STORING BINARY PATTERNS IN TWO-DIMENSIONAL NETWORKS OF NONLINEAR SYSTEMS

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Abstract— *In this paper, we investigate synchronization and cluster formation phenomena in two-dimensional arrays of locally interconnected chaotic circuits. We report the existence of an abundance of attractors, for which each cell stores a binary information. We describe a simple method for storing binary patterns in the network. We also address the question which patterns can be successfully stored in the network and discuss problems of pattern stability and influence of parameter mismatch.*

I. INTRODUCTION

One of the theories explaining the functionality of the brain relies on the dynamical representations. Construction of patterns of brain activity constitutes the key to understanding of various phenomena including perception, memory, attention etc. [2]. Many different kinds of artificial neural networks have been proposed to mimic such functionality [1], [3], [4]. Also special types of information processing can be obtained using Cellular Nonlinear Networks [5], [6]. In this paper we combine two aspects - chaotic unit cells and abundance of existing attractors to obtain binary pattern storage.

After introduction of the dynamical array in section III we study the problem of existence of many attractors, corresponding to binary patterns. In section IV we describe how to force the network to store a given binary pattern. We also show examples of patterns, which cannot be stored and attempt to characterize those patterns. In section V we investigate stability of patterns, the size of their basins of attraction and influence of parameters mismatch. Finally in section VI, we show two examples of behavior of larger networks and discuss the influence of network size on the ability of the network to store binary patterns.

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II. DYNAMICS OF THE NETWORK

Let us consider a two–dimensional array composed of simple third–order nonlinear systems (Chua’s circuits). The dynamics of an $n \times m$ array can be described by

$$\begin{aligned} C_2\dot{x}_{i,j} &= G(z_{i,j} - x_{i,j}) - y_{i,j} + \sum_{(k,l)\in N_{i,j}} G_1(x_{k,l} - x_{i,j}), \\ L\dot{y}_{i,j} &= x_{i,j}, \\ C_1\dot{z}_{i,j} &= G(x_{i,j} - z_{i,j}) - f(z_{i,j}), \end{aligned} \tag{1}$$

where $i = 0, 1, 2, \ldots, n-1$, $j = 0, 1, 2, \ldots, m-1$ and f is a five–segment piecewise linear function:

$$\begin{aligned} f(z) = & m_2 z + 0.5 \cdot (m_1 - m_2)(|z+b_2| - |z-b_2|) + \\ & 0.5 \cdot (m_0 - m_1)(|z+b_1| - |z-b_1|). \end{aligned} \tag{2}$$

$x_{i,j}$ and $z_{i,j}$ denote the voltages across the capacitors C_2 and C_1 respectively, and $y_{i,j}$ is the current through the inductor L in the cell (i, j) (i.e. belonging to the ith column and jth row – see Fig. 1). $N_{i,j}$ de-

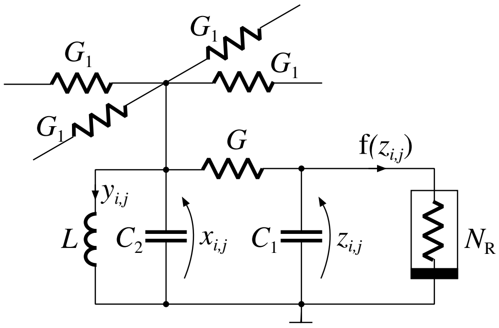

Fig. 1. A third order circuit coupled with its neighbors by means of conductances G_1

notes the neighborhood of the cell (i, j), i.e. a set of cells directly connected with the cell (i, j). We consider the case when each cell is connected with its four nearest neighbors (i.e. $N_{i,j} = \{((i+1) \bmod n, j), ((i-1) \bmod n, j), (i, (j+1) \bmod m), (i, (j-1) \bmod m)\}$) by means of conductances G_1 (here $G_1 = 20$). In our study we use parameter values for which an isolated circuit generates the "double scroll" chaotic attractor: $C_1 = 1/9, C_2 = 1, L = 1/7, G = 0.7, m_0 = -0.8, m_1 = -0.5, m_2 = 0.8, b_1 = 1, b_2 = 2$.

III. Existence of many attractors

Let us consider the network composed of $n \cdot m = 10 \cdot 10$ cells. To make a classification of steady states of the network we have run a number of simulations starting the network with random initial conditions. Four examples are shown in Figs. 2–5. In each case the network converges to a limit set (steady state). To show the state of the whole network, we plot a snapshot using shades of gray to represent the value of z variable in each cell. We plot also projections of the system trajectory onto chosen sub-spaces. Projection onto the plane $(z_{i,j}, y_{i,j})$ shows a trajectory of a given cell, while the projection onto the plane $(z_{k,l}, z_{i,j})$ indicates the synchronization between two cells.

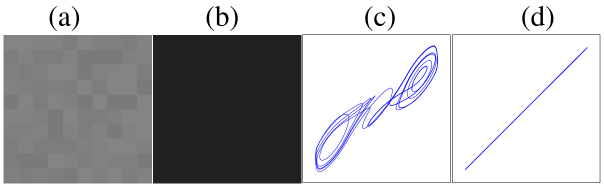

Fig. 2. (a) Initial state ($t = 0$), (b) Steady state ($t = 300$); Plots of variables in the steady state: (c) $y_{0,0}$ versus $z_{0,0}$, (d) $z_{4,6}$ versus $z_{0,0}$.

For the first example snapshots of the initial state and the steady state are shown in Fig. 2(a,b). A uniform coloring for the whole network in Fig. 2(b) indicates that all cells are synchronized. This is confirmed by plotting projection of the trajectory onto the plane $(z_{k,l}, z_{i,j})$ for two distant cells (k, l) and (i, j) (see Fig. 2(d)). Trajectories of individual cells form double–scroll attractors (Fig. 2(c)). Fully synchronized state is observed most frequently when the network is started from random initial conditions, which indicates that its basin of attraction is large.

In the second example, in the steady state there are two clusters of cells oscillating synchronously. This corresponds to groups of light and dark squares in Fig. 3(b). Oscillations generated by cells belonging to different clusters are shown in Fig. 3(c,d). The cells in different clusters operate in distinct regions of the $\mathbb{R}^3$ space (for cells in one cluster $z > 0$, while for the second one $z < 0$). Very good synchronization between the cells within clusters is shown in Fig. 3(e). In the steady state the behavior of the network is periodic.

Fig. 4 shows an example where one cluster is much smaller than the other. Clusters have sizes 17 and 83, respectively. Another important difference is that the network in the steady state oscillates chaotically (see Fig. 4(c,d)). There is very good synchronization between cells belonging to each cluster (Fig. 4(e)).

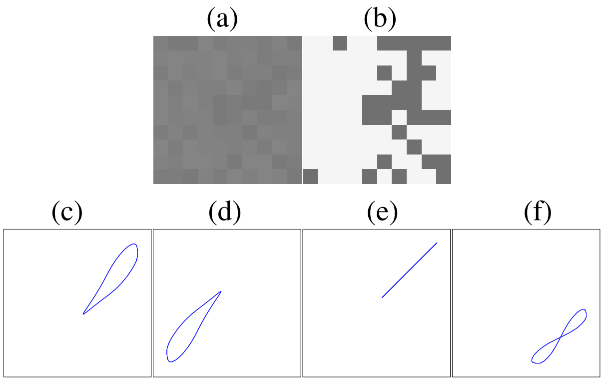

Fig. 3. (a) snapshot at $t = 0$, (b) snapshot at $t = 300$; plots in the steady state: (c) $y_{0,0} - z_{0,0}$, (d) $y_{4,4} - z_{4,4}$, (e) $z_{4,6} - z_{0,0}$, (f) $z_{4,4} - z_{0,0}$.

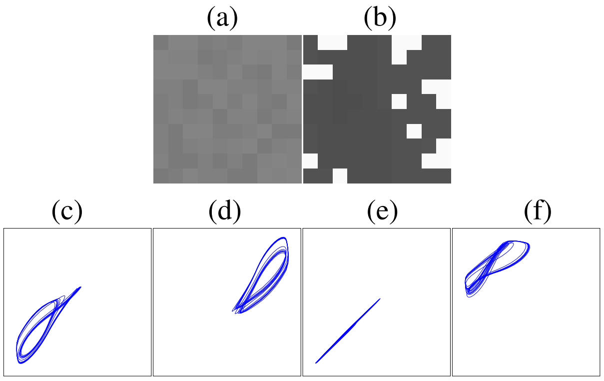

Fig. 4. (a) Initial state ($t = 0$), (b) steady state ($t = 300$); plots in the steady state: (c) $y_{0,0} - z_{0,0}$, (d) $y_{6,0} - z_{6,0}$, (e) $z_{4,4} - z_{0,0}$, (f) $z_{6,0} - z_{0,0}$.

The last example shows pattern switching phenomena. The network started with random initial conditions for time $t \in [20, 45]$ displays a pattern with cluster sizes 94 and 6 (Fig. 5(a)). This structure is however not stable. At $t \approx 45$ the pattern changes. Most of the cells from the larger cluster leave the region $z > 0$ and a pattern with clusters of size 13 and 87 emerges (Fig. 5 (b)). This last pattern is stable. After very long integration time is still persists in the network. The cells within a cluster are not fully synchronized. Sometimes small bursts can be seen (Fig. 5(c)), but all the time the cluster pattern is clearly visible.

Among an abundance of observed attractors most frequent is the state of full synchronization. Other attractors are characterized by two clusters of cells operating in distinct regions of the $\mathbb{R}^3$ space. In some sense this attractors can be regarded as binary patterns. If a cell operates in the region $z > 0$ (or $z < 0$) we say it corresponds to binary "1" (or "0"). In some

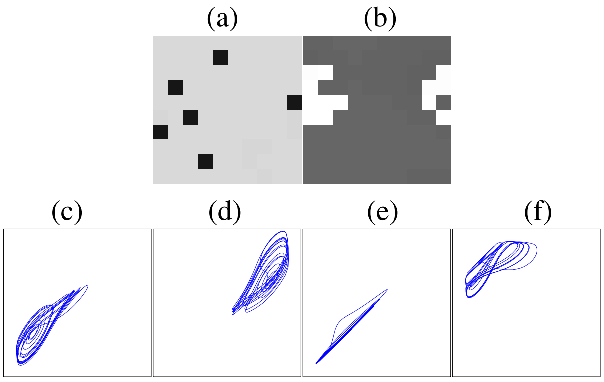

Fig. 5. (a) Initial state ($t = 10.5$) and (b) steady state ($t = 300$); plots in the steady state: (c) $y_{0,0} - z_{0,0}$, (d) $y_{1,5} - z_{1,5}$, (e) $z_{5,5} - z_{0,0}$, (f) $z_{1,5} - z_{0,0}$.

cases patterns are not stable.

IV. Storing patterns

An important question, which arises in this context is how can we force the network to display a given pattern. We test a very simple approach. First, we choose a point (x, y, z) on the double-scroll attractor positioned far from the hyperplane $z = 0$ and satisfying the condition $z > 0$. In cells, which we want to code as binary "1" we set $(x_{i,j}, y_{i,j}, z_{i,j}) = (x, y, z)$ as an initial condition. For other cells we set $(x_{i,j}, y_{i,j}, z_{i,j}) = (-x, -y, -z)$. It appears that in this simple way we can force the network to store a given binary pattern. An example is shown in Fig. 6.

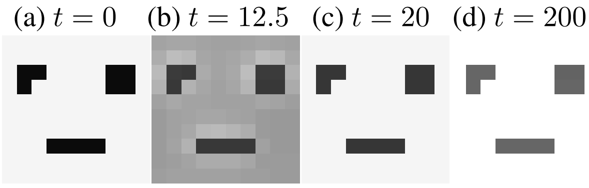

Fig. 6. Storing a pattern

Since the initial state (Fig. 6(a)) does not belong to the attractor corresponding to the binary pattern stored, we observe transient oscillations (see shades of gray in Fig. 6(b)). After some time the network converges to the attractor. Snapshots taken at $t = 20$ and $t = 200$ confirm that in the steady state the cells in each cluster oscillate synchronously.

In Fig. 7, we show four other examples of patterns that were successfully stored in the network.

There are some patterns which are not stable. Two examples are shown in Fig. 7(e,f). If we try to impose these patterns using the method described above the system displays them for some time, but eventually escapes to a stable attractor – in both cases the trajectory is attracted to the steady state with all cells synchronized. It seems that patterns for which one of the clusters is very small are unstable. Further analysis is necessary to characterize the class of unstable patterns.

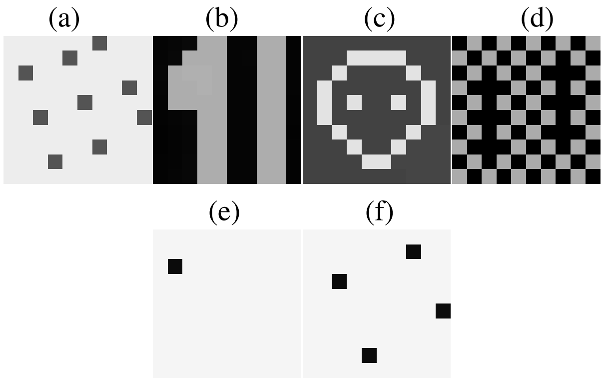

Fig. 7. First row – stored patterns (snapshot at $t = 40$); second row – unstable patterns, impossible to store

V. Stability of patterns

In order to study stability of binary patterns, we carry out two different tests.

In the first test we modify the network parameters. All parameters of cells are disturbed by a random deviation of maximum amplitude 0.01% of nominal value. Two snapshots taken at the steady state are shown in Fig. 8. Initially a binary pattern is formed. Later, the cells within clusters are not fully synchronized (different shades of grey) but still the cluster structure is clearly visible.

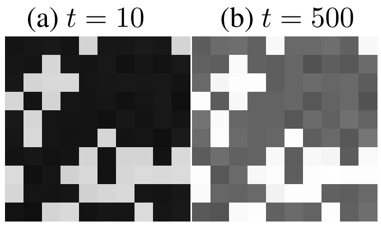

Fig. 8. Behavior of network with nonuniform cells.

In the second test we disturb all variables in the network displaying a binary pattern by adding a random value of a small amplitude. In this way we can test the size of the basin of attraction of the corresponding attractor. Results for the pattern form Fig. 6(d) are shown in Fig. 9. The maximum amplitude of perturbation was 0.5 (Fig. 9(a)) and 1.0 (Fig. 9(c,e)). In two cases the pattern was recovered in a correct way (Fig. 9(b,d)), while in the last case one bit was detected with error (Fig. 9(f)).

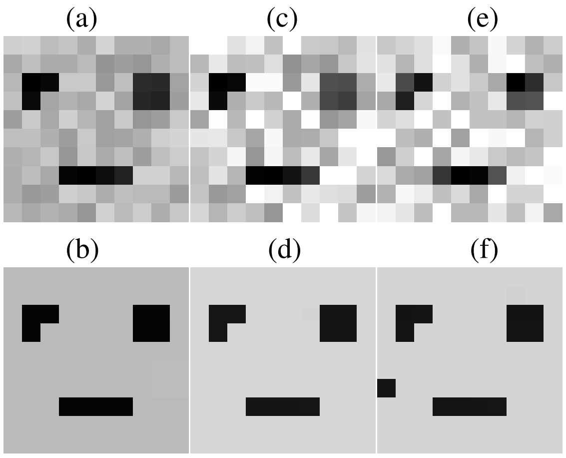

Fig. 9. Stability of patterns, (a), (c), (e) disturbed patterns, (b), (d), (f) recovered patterns

VI. Larger networks

Let us now consider two examples of larger networks. The behavior of a 20×20 network started with random initial conditions is shown in Fig. 10. The binary pattern visible in Fig. 10(b) evolves (see Fig. 10(c)) and around $t = 200$ the final pattern emerges (Fig. 10(d)). This pattern persists even for $t \leq 5000$. Although the binary pattern is stable there is no full synchronization between the cells in the clusters – waves traveling through the network are visible as different shades of gray for cells belonging to a given cluster.

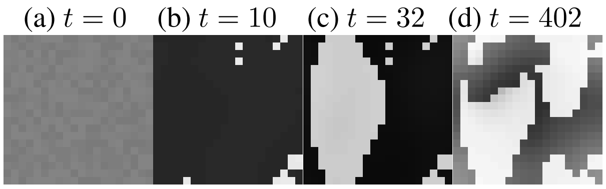

Fig. 10. Dynamics of a network of 20×20 cells

As a last example we show simulations of the network composed of 100×100 cells. In this case the network started from random initial conditions after $t > 4$ displays a binary pattern (Fig. 11(b)). This pattern is however not stable. At $t = 18$ a circular wave appears and the behavior becomes disorganized. Snapshots taken at $t = 100$ and $t = 500$ are shown in Fig. 11(c,d). The mode of pattern variation is typical for large networks.

These two simulations show that the property of storing binary patterns depends on the network size. It seems that the number of stable patterns is smaller for large networks. For very large networks binary patterns are not stable and more complex behavior is observed.

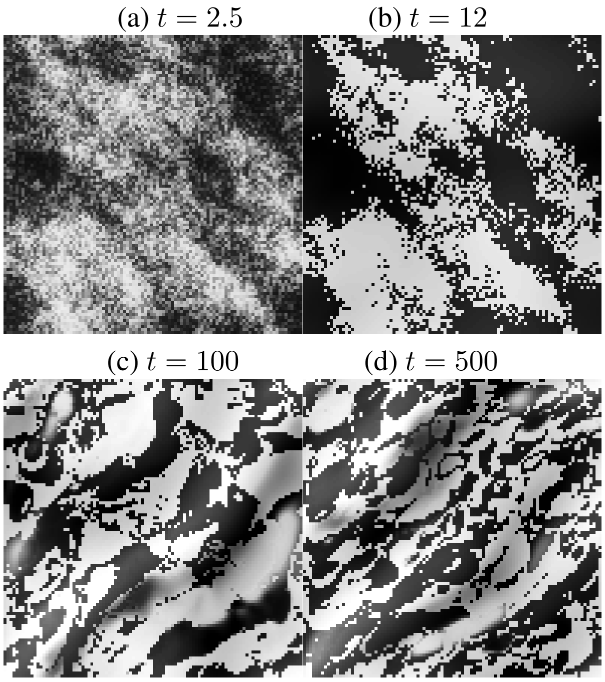

Fig. 11. Behavior of a network of 100×100 cells

VII. Conclusions

Arrays of locally coupled chaotic circuits show an abundance of pattern formation phenomena. Using position of the attractors in the phase space it is possible to give a binary description to such patterns. We have investigated the formation of binary patterns in arrays of Chua's circuits coupled via a regular resistive grid. Further we have proposed a simple method for obtaining a desired binary pattern by appropriate choice of initial states of the network. Stability of patterns and influence of non-uniformity and size of the networks have also been addressed.

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