Enhanced Differential Chaos Shift Keying Using Symbolic Dynamics

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***Abstract*-In this paper we propose an enhanced version of the DCSK (Differential Chaos Shift Keying) scheme where the chaotic carrier is exploited for conveying useful information. This is achieved by means of a pseudo-chaotic encoder which spreads the input sequence, approximating the dynamics of the Bernoulli shift. The information encoded in the chaotic carrier is retrieved by means of standard maximum-likelihood detection methods.**

I. Introduction

Among the several proposed chaos-based communication schemes DCSK (Differential Chaos Shift Keying) [1], [2], [3] exhibits one of the best BER performances and it has been shown to be particularly robust against multipath fading [4]. In DCSK, for each symbol period, a portion of chaotic waveform (reference signal) is transmitted followed by its inverted or non-inverted copy (information-bearing signal) depending on the bit of information. At the receiver, the information is extracted by means of differentially coherent demodulation, that is by correlating the information-bearing part of the signal with the reference. However, as pointed out in [5], part of the information associated with the chaotic carrier remains unexploited. We emphasize that a chaotic system may be seen as an information source, which reveals more and more information about its initial state during the evolution in time [6]. Actually, the entropy associated with a chaotic signal can be measured by introducing the formalism of symbolic dynamics [7]. For piecewise linear Markov (PWLM) maps there exists a natural discretization which allows to define a one-to-one relationship between any initial condition and an (infinite) symbolic sequence [8].

In DCSK the total channel capacity is shared between the payload bits transmitted and the symbolic sequence, which is also transmitted (in a hidden way), associated with the chaotic carrier. Furthermore, the two sources are clearly independent [5]. In this work we propose an enhanced version of DCSK, which we call SD-DCSK, exploiting the symbolic dynamics (SD) associated with the chaotic carrier. This is obtained by replacing the chaotic generator used in DCSK with a pseudo-chaotic (PC) encoder [9], which generates a chaotic signal from the original input sequence. In practice, the pseudo-chaotic encoder is realized with a convolutional-like encoder followed by a DAC (digital/analog converter), approximating the iterates of the chaotic Bernoulli shift [10]. We show that the information encoded in the pseudo-chaotic carrier can be extracted efficiently using standard Viterbi detection, with an appropriate metric definition. Also, the corresponding trellis exhibits an interesting scalability property deriving directly from the symbolic dynamics approach. This property allows to use a large number of levels at the transmitter while decoding with reduced complexity, adding a degree of freedom in terms of the receiver design.

In summary, the SD-DCSK scheme allows the creation of an auxiliary communication channel, parallel to the DCSK one. This can be used to increase the data rate of the system by transmitting independent information. If instead the same binary stream is sent as for DCSK, it can be employed for error-correction purposes [11].

II. SD-DCSK

The basic idea behind the SD-DCSK scheme is to replace the chaotic generator present in the DCSK scheme [2] with a pseudo-chaotic encoder, as illustrated schematically in Fig. 1. We denoted with $m_1(k)$ the binary stream to be modulated according to the original DCSK scheme. On the other hand, the input sequence $m_2(k)$ is used to generate the pseudo-chaotic carrier. Note that the inputs m_j ($j = 1, 2$) are independent from each other, although the same sequence can be transmitted in both channels in order to increase the reliability of the communication. The only assumption that is made is on $m_2(k)$ to be an i.i.d. (independent identically distributed) sequence, as discussed later.

For simplicity in this work we consider a baseband system and, for illustrative purposes only, a simple instance of phase modulation meant to maintain the transmitted energy per bit constant.

A. Pseudo-Chaotic Encoder

The PC-encoder performs a spreading of the input sequence $m_2(k)$, mimicking the chaotic dynamics of the Bernoulli shift map, defined by [10]:

$$x_{k+1} = 2x_k \mod 1 \tag{1}$$

whose graph is shown in Fig. 2. The state x can be represented as a binary expansion:

$$x = 0.b_1 b_2 b_3 \ldots \equiv \sum_{j=1}^{\infty} 2^{-j} b_j \tag{2}$$

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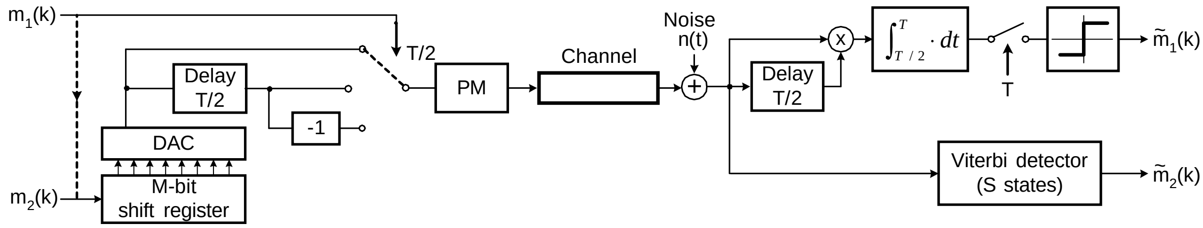

Fig. 1. Block diagram of the SD-DCSK scheme. For forward error-correction applications: $m_2(k) \equiv m_1(k)$.

where each of the bits b_j is either a "0" or a "1", and $x \in I = [0,1]$. The successive iterates of x are obtained by moving the separating point one position to the right (multiplication by 2) and setting to zero the integer digit (modulo 1 operation). Hence, digits which are initially far to the right of the separating point, thus having only a very slight influence on the value of x, eventually become the first fractional digit. The information is encoded by associating the symbol "0" to the subinterval $I_0 = [0, 0.5)$ and the symbol "1" to $I_1 = [0.5, 1]$, as shown in Fig. 2.

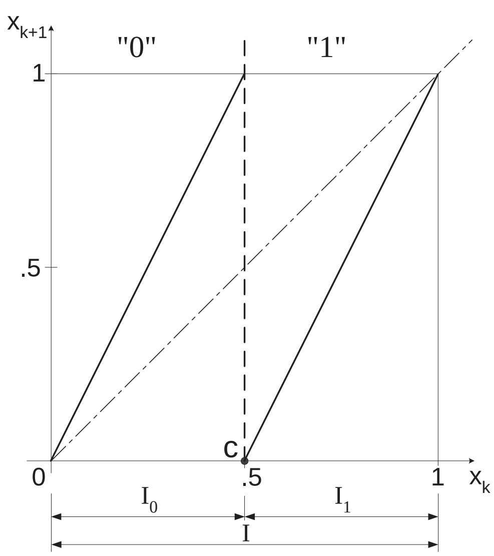

Fig. 2. The Bernoulli shift map. The invariant interval $I = [0,1]$ is partitioned with respect to $c = 0.5$. The subintervals I_0 and I_1 are assigned the binary symbols "0" and "1", respectively.

In this work the Bernoulli shift process is approximated by means of an M-bit shift register followed by a DAC, as illustrated in Fig. 1. Correspondingly, the generic state x_l (with $l = 1, 2, \ldots, 2^M$) can be expressed as:

$$x_l = 0.b_1 b_2 \ldots b_M \equiv \sum_{j=1}^{M} 2^{-j} b_j \tag{3}$$

where b_1 and b_M represent the MSB (most significant bit) and the LSB (least significant bit), respectively. The shift operation corresponds to a multiplication by a factor 2, while the modulo 1 operation is realized by discarding the shifted MSB at each step. The shift register is fed with the input sequence $m_2(k)$, which we assume to be i.i.d.[1] At each step the most recent bit of information is assigned the LSB position while the old MSB is discarded. Due to the finite length of the shift register, the dynamics of the Bernoulli shift can only be approximated. In particular, for an M-bit shift register the quantization error ε is bounded from above by $\varepsilon(M) < 2^{-M}$, which tends to zero for $M \to \infty$.

¿From the viewpoint of information theory the shift register structure implementing the Bernoulli shift may be seen as a form of convolutional coding [12]. The memory of the structure is represented by the shift register which stores the last M input bits. Each input bit causes an output of M bits; thus, the overall code rate is $1/M$.

In general, the shift register implementing the Bernoulli shift map may be followed by a transformation unit for generating more complex chaotic maps. For example, a Gray/Binary converter can be used to generate the tent map.

B. SD-DCSK Signal

In this work we consider DCSK for the case $L = 1$, which means that for each symbol period T one pseudo-chaotic iterate is generated [2]. In order to maintain the transmitted energy per bit constant we adopt a simple phase modulation (PM) scheme. Namely, given the k-th PC-iterate, x_k, we define the corresponding symbol s_k as follows: $s_k = [\cos(\varphi_k), \sin(\varphi_k)]$ where $\varphi_k = 2\pi x_k + \varphi_0$, and $\varphi_0 = 2\pi/2^{M+1}$ is a phase offset for facilitating the definition of the decision boundaries. The corresponding signal-space diagram is shown in Fig. 3. Note that with these notations $\varphi_k \in [0, 2\pi]$. Thus, the invariant interval $I = [0,1]$ of the Bernoulli shift, with its definition of symbolic dynamics, maps to the unit circle in the signal space.

Then, in the first half $[0, T/2]$ of the symbol period we transmit the reference vector:

$$Y = [Y_1, Y_2] = \left[\sqrt{\frac{E_b}{2}}\cos(\varphi_k), \sqrt{\frac{E_b}{2}}\sin(\varphi_k)\right]$$

followed by $\pm Y$ in the second half $[T/2, T]$, depending on the input bit $m_1(k)$. An example of transmitted SD-DCSK

[1] In practice this may be achieved by inserting a data compression and/or a data scrambling block in front of the shift register.

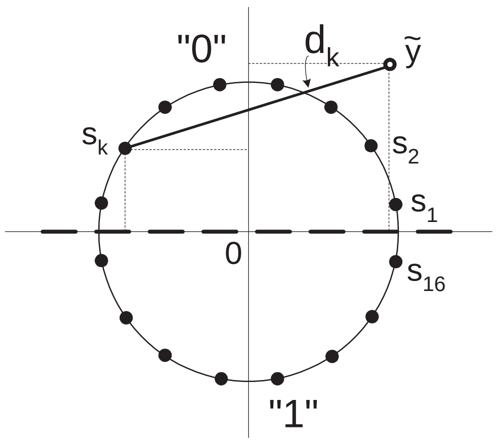

Fig. 3. Signal-space diagram for the SD-DCSK transmitted signal (for $M = 4$) illustrating the geometrical meaning of the branch metric d_k. Note also that the arc $[0, \pi)$ is associated with the symbol "0", while $[\pi, 2\pi)$ corresponds to the symbol "1".

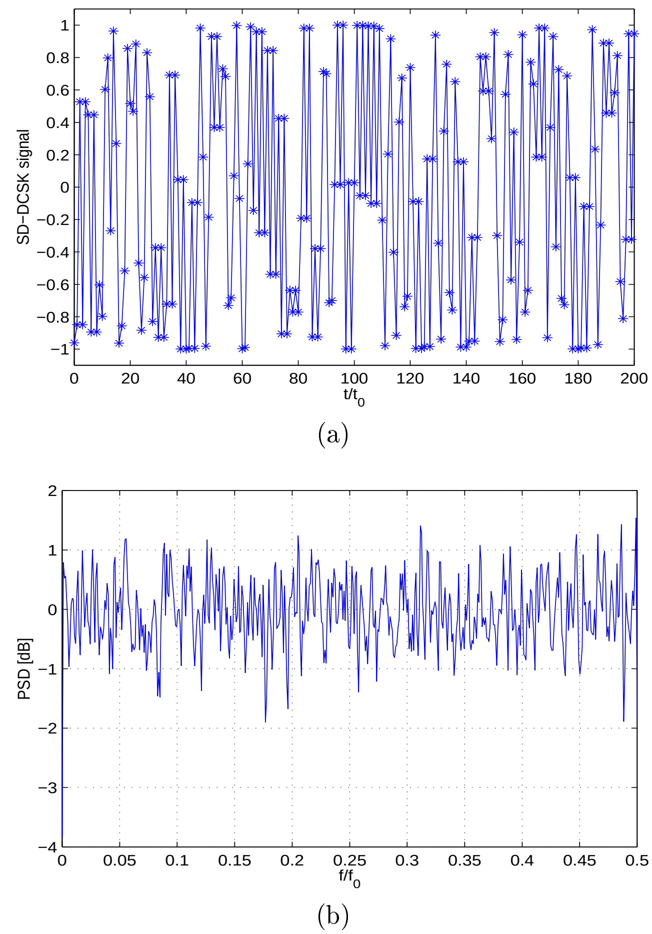

Fig. 4. (a) Transmitted SD-DCSK signal (for $M = 10$) and (b) corresponding PSD (power spectral density). The plots refer to normalized time t/t_0 and normalized frequency f/f_0, where $t_0 = 1/f_0$ is the sampling period.

signal and its spectrum are illustrated in Fig. 4, in normalized units. Note that for practical applications (and assuming $m_2(k)$ to be an i.i.d. sequence) the pseudo-chaotic signal can be considered equivalent to the one produced by a real chaotic generator.

C. SD-DCSK Demodulation

The scheme proposed allows the creation of an auxiliary communication channel to the DCSK one, exploiting the symbolic dynamics associated with the chaotic carrier. As far as the demodulation is concerned, the two channels can be considered independent from each other.

D. DCSK

Referring to Fig. 1, the DCSK demodulation is carried out as usual, *i.e.* by correlating the information-bearing part of the signal ($t \in [T/2, T]$) with the reference ($t \in [0, T/2]$), sampling the correlator output according to the symbol period T, and inferring on the symbol received by means of a threshold detector [2].

E. SD Decoder

The information encoded in the pseudo-chaotic carrier can be extracted from the received signal by considering its reference part. Further methods may be applied to take into account the redundancy due to the information-bearing part of the signal, but this is outside the scope of the present paper.

Threshold Detection

In the simplest case the pseudo-chaotically encoded data can be detected by a phase discriminator detecting whether $\varphi_k \in [0, \pi]$ or $[\pi, 2\pi]$, respectively (see Fig. 3). This method is not very effective as it relies solely on the sign of the sample Y_2. In addition, because of the signal-space diagram associated with the PC-encoder, symbols such that $\sin(\varphi_k) \approx 0$ will cause an error also for relatively low noise levels.

Maximum-Likelihood Detection

By assuming the input $m_2(k)$ to be an i.i.d. sequence, the optimal decoder for the PC-encoded signal is represented by a trellis matched to the dynamics of the Bernoulli shift, seen as convolutional encoder. In fact, we recall that every PWLM map admits a representation as a topological Markov chain [8]. In this work we consider soft Viterbi decoding and for illustrating the branch metric computations we consider the normalized vector $y = \sqrt{2/E_b}Y$. At each step, the input of the Viterbi algorithm is a vector $\tilde{y} = [\tilde{y_1}, \tilde{y_2}]$ representing the received symbol affected by noise. By assuming that each sample is perturbed by an independent Gaussian variable (AWGN) it can be shown that the observation probability of receiving $\tilde{y}$, if the symbol $s_k = [s_{k1}, s_{k2}]$ was sent, is proportional to $e^{\left((\tilde{y_1}-s_{k1})^2+(\tilde{y_2}-s_{k2})^2\right)/2\sigma_n^2}$ where σ_n is the noise variance. This in terms of logarithms, according to the usual formulation of the Viterbi algorithm, translates into the branch metric:

$$d_k = \sqrt{(\tilde{y_1} - s_{k1})^2 + (\tilde{y_2} - s_{k2})^2} \qquad (4)$$

whose geometrical interpretation is shown in Fig. 3.

Note that the same PC-encoded signal can be decoded

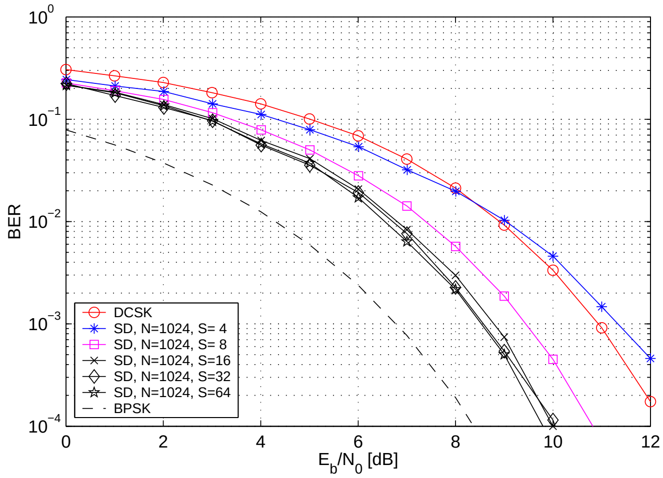

Fig. 5. BER performance: DCSK versus pseudo-chaotic modulation. Note the dependence on the number of states S in the Viterbi detector and the high error probability associated with the threshold discriminator (THR).

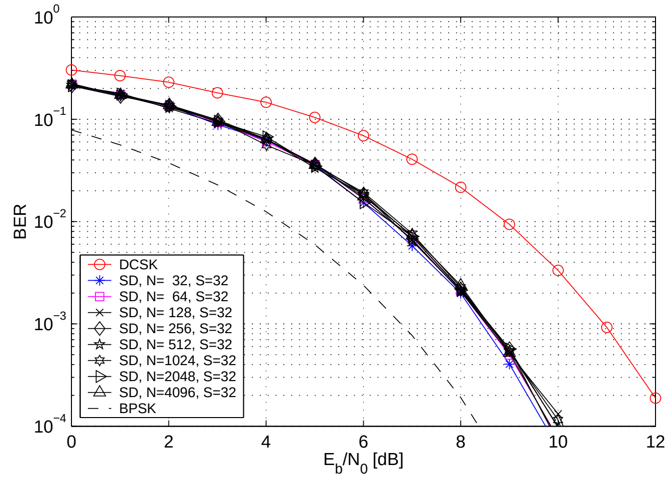

Fig. 6. BER dependence on the number N of transmitter levels, for a fixed complexity ($S = 32$) of the Viterbi detector. Note that the performance is basically independent from the number of transmitter levels.

by Viterbi detectors with different number of states. This is a direct consequence of the symbolic dynamic approach used in this work for encoding/decoding information in the pseudo-chaotic carrier. Namely, given the number $N(= 2^M)$ of transmitter levels, the received PC-signal can be decoded with Viterbi detectors with $S = 2, 4, 8 \dots, N$ states. This requires a slight modification in the branch metric computation; in particular, in this work we compute the observation probabilities based on the transmitter level "closest" to the received signal. We emphasize that the scalability property allows to perform a high spreading at the transmitter (large N) such that the PC-iterates reproduce with very good approximation the dynamics of the Bernoulli shift, while decoding with reduced complexity (small S). Of course, the performance of the Viterbi detector depends on the number of states, as discussed in the next section.

III. BER Performance

The results of our analysis are presented in terms of BER probability versus the ratio E_b/N_0 expressed in dB, where E_b is the energy per user bit—which coincides for $m_1(k)$ and $m_2(k)$—and N_0 is the single-sided spectral noise density. We consider here the interference on the channel to be just AWGN (additive white Gaussian noise).

Fig. 5 shows the simulation results for the auxiliary communication channel associated with the pseudo-chaotic carrier, versus DCSK. We observe that the simple phase discriminator results in a high error probability, deriving from the structure of the signal-space diagram associated with the PC-encoding (see Fig. 3). However, soft Viterbi decoding results in a good BER performance, that is significantly better than DCSK (even though the energy associated with the information bearing part of the signal is not utilized). Note the BER dependence on the number S of states in the Viterbi detector, confirming the scalability property discussed above. Also, we note a saturation effect in the performance when increasing the number of states above a certain number (in the example $S = 32$).

On the other hand, Fig. 6 illustrates the BER dependence on the number N of transmitter levels, for a given complexity, *i.e.* a given number of states of the Viterbi detector. Note that the performance is basically related only to the number S of states. This allows to define arbitrarily many transmitter levels, in order to minimize the quantization error, without compromising the overall performance of the auxiliary channel.

IV. Conclusions

In this work we have proposed an enhanced version of the DCSK scheme which takes advantage of the symbolic sequence associated with the chaotic carrier for conveying useful information. This allows the creation of an auxiliary communication channel to the DCSK one which can be used to increase the overall data rate and/or for error-correction purposes.

Acknowledgments

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