PATTERN CODING IN A ONE–DIMENSIONAL ARRAY OF COUPLED CHAOTIC OSCILLATORS

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Abstract— In this paper we report the existence of a very large number of patterns corresponding to different chaotic attractors in a one–dimensional array of coupled chaotic circuits. We estimate the number of patterns that can be stored in the network for different network size.

I. INTRODUCTION

In nature structures composed of individual simple subsystems and wide-spread. Specific examples come from biology and medicine (tissues of living organisms) physics and chemistry (matter composed of atoms), etc. Properties of such systems depend on properties of individual subsystems and the way they are coupled together. Various models describing behavior of interconnections of a large number of simple systems have been proposed by scientists. Among them lattice models, exhibiting various types of collective behavior play an important role [1, 2, 3, 4, 5].

Among various types of collective dynamics one can observe many types of spatial, temporal or spatio-temporal ordered structures referred to as self-organization [1] or "pattern formation". "Organized" behavior is usually linked with coherent (synchronized) behavior of a number of subsystems in the network. Organized spatio-temporal behavior includes propagation of waves including solitons and autowaves, target waves, spiral waves and traveling wavefronts.

In our previous works we studied cooperative behavior in one- and two-dimensional arrays of Chua's circuits with resistive coupling between the cells [6, 7]. In the present study we investigate steady-state behavior observed in a ring (one–dimensional array with connected ends) of coupled Chua's chaotic circuits. In experiments we use so-called balanced cells in which a self-coupling term has been introduced in each cell enabling simultaneous development of synchronized chaotic motion in all cells. Using computer experiments we have confirmed the existence of a very large number of stable final states depending on the connection strength and initial conditions applied in the individual cells. Existence of various synchronized states is studied experimentally. These final states can be coded using a binary alphabet.

II. EXPERIMENTAL SETUP

Let us consider a one–dimensional lattice of simple third–order electronic oscillators (Chua's circuits). The oscillators are coupled bi-directionally by means of two resistors cross-connected between the capacitors C_1 and C_2 of the neighboring cells (compare Fig. 1a).

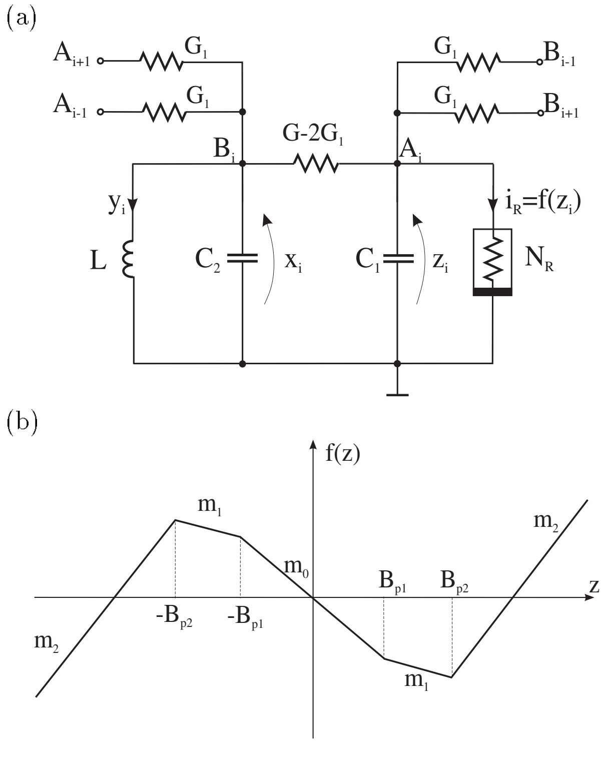

Figure 1: (a) A one–dimensional array of simple third–order oscillators, (b) A five–segment piecewise linear function.

Every cell is connected with two nearest neighbors. The first and the last cells are also connected and the lattice forms a ring. The dynamics of the lattice composed of n cells can be described by the following set

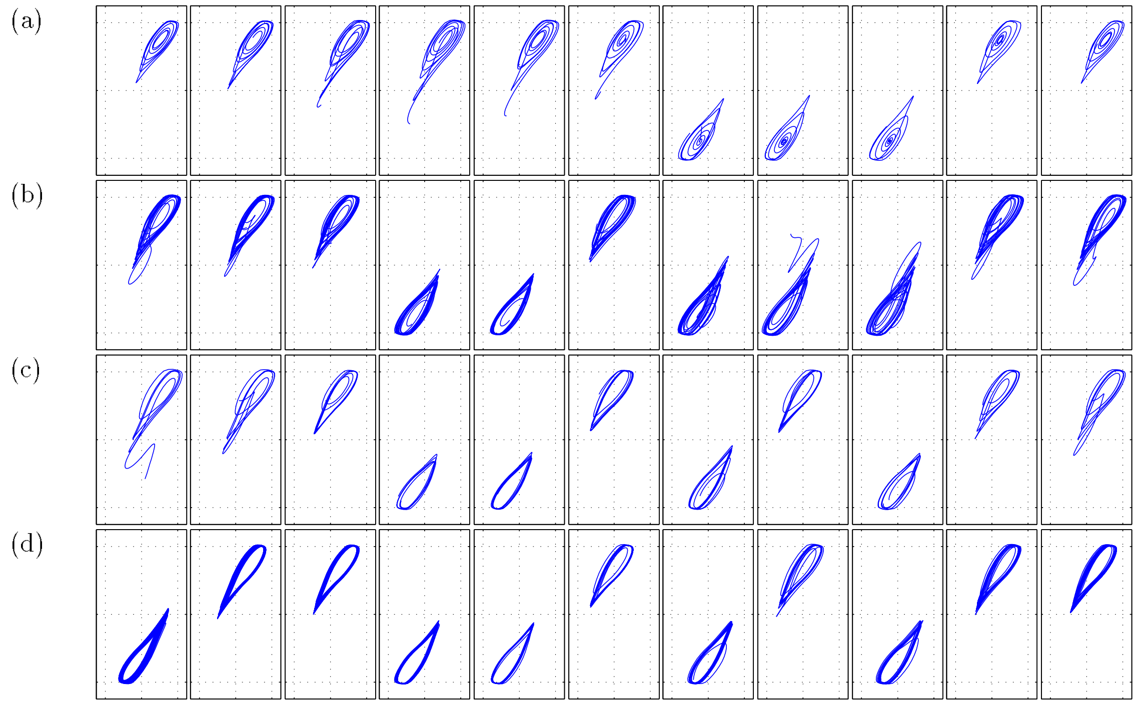

Figure 2: Switching of patterns in an array composed of 11 circuits (a) $T \in [3, 24]$, (b) $T \in [26, 73]$, (c) $T \in [73, 89]$, (d) $T \in [89, 189]$ — final steady–state pattern.

of ordinary differential equations [6], [7]:

$$\begin{aligned} C_2\dot{x}_i &= -y_i + (G - 2G_1)(z_i - x_i) + \\ &\quad + G_1(z_{i-1} - x_i) + G_1(z_{i+1} - x_i), \\ L\dot{y}_i &= x_i, \\ C_1\dot{z}_i &= (G - 2G_1)(x_i - z_i) - f(z_i) + \\ &\quad + G_1(x_{i-1} - z_i) + G_1(x_{i+1} - z_i), \end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, n$ and we use the following boundary conditions $x_0 := x_n$, $z_0 := z_n$, $x_{n+1} := x_1$ and $z_{n+1} := z_1$ and f is a five–segment piecewise linear function (compare Fig. 1b):

$$\begin{aligned} f(z) &= m_2 z + \frac{1}{2}(m_1 - m_2)(|z + B_{p_2}| - |z - B_{p_2}|) \\ &\quad + \frac{1}{2}(m_0 - m_1)(|z + B_{p_1}| - |z - B_{p_1}|). \end{aligned} \tag{2}$$

As in our previous studies we use typical parameter values for which an isolated Chua's circuit generates chaotic oscillations — the "double scroll" attractor:

$$\begin{array}{lll} C_1 = 1/9F, & C_2 = 1F, & L = 1/7H, \\ G = 0.7S, & m_0 = -0.8, & m_1 = -0.5, \\ m_2 = 0.8, & B_{p_1} = 1, & B_{p_2} = 2. \end{array} \tag{3}$$

For these parameter values together with a chaotic attractor there exist periodic orbit with a large amplitude. In the experiments we have considered the uniform coupling G_1.

III. SIMULATION RESULTS

Let us consider the network composed of $n = 11$ circuits connected using a coupling $G_1 = 0.2$.

We have performed a very large number of experiments, starting this network with small random initial conditions of amplitude 0.01. A typical result of such experiment is shown in Fig. 2.

In the steady–state the network behaves chaotically, with some cells developing Roessler–type attractors in the upper and some in the lower half–space (compare Fig. 2(d)).

Such a state of the network when some cells operate in the upper half–space, while others in the lower half–space will be called a pattern. With each patterns we associate the sequence of 0's and 1's in such a way that if the ith cell operates in the upper (lower) half–space then we set the ith element of the sequence to 1 (0). Hence the sequence corresponding to pattern from Fig. 2(d) is 01100101011.

One can observe very interesting phenomena of pattern switching (compare Fig. 2). It seems that patterns including several adjacent cells operating in the same half–space are less stable. After certain time they tend to convert to a pattern with shorter clusters of cells operating in the same half–space. Usually the neighboring cells operating in the same half–space are almost synchronized, hence we will refer to such cells as "coherent" cells.

We think that the pattern switching phenomena can be explained in the following way. Usually if the cluster of "coherent" cells is large, the corresponding attractors are larger and thicker than in a case when the cluster size is small (< 3). After some time the trajectory of one of the cells from the large "coherent" cluster enters the second half–space. The pattern is switched to a pattern with shorter "coherent clusters", the attractors of individual cells become thinner and smaller and this new pattern is more stable than the initial one.

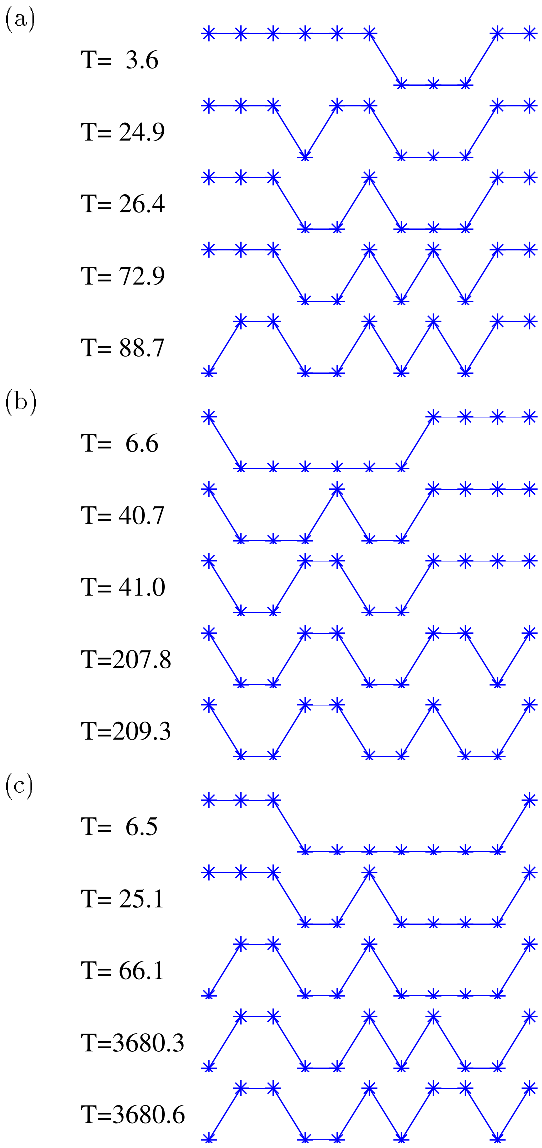

Figure 3: Switching of patterns in an array composed of $n = 11$ circuits. In all examples the final pattern is composed of single–cell and double–cell clusters only.

Three examples of pattern switchings are shown in Fig 3. For each pattern switching we also record its time instant, which is printed to the left to the pattern. In all examples after 5 pattern switchings the steady–state pattern, which does not contain clusters with size larger than 2, is obtained. The number of pattern switchings before the steady–state can be very different, but usually if we start from very small initial conditions it is smaller than 10.

In Fig. 4 we show another example of pattern switching. This time the number of pattern switching before the steady–state is 7. See that the last pattern switching occurs after a very long time $T > 3400$. Cluster composed of three cells converts to three single–cell clusters. In this case in the steady–state we also observe a cluster composed of three "coherent" cells (cells 9–11).

During our experiments we have observed many examples of clusters composed of three or more cells, that seemed to be stable, but usually after a long time the pattern evolved into a pattern composed of shorter clusters.

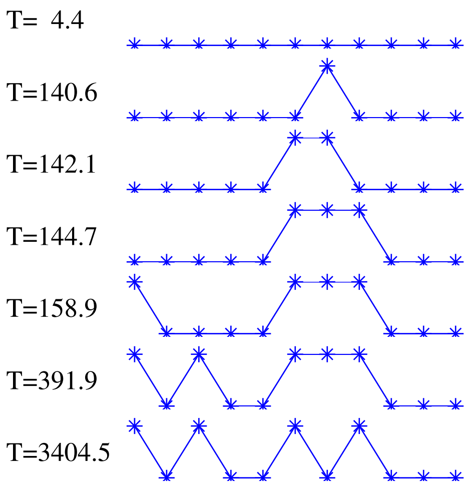

Figure 4: Switching of patterns in an array composed of $n = 11$ circuits. In the steady–state the pattern contains the cluster composed of three cells.

From the experiments we conclude that all patterns not containing clusters with size larger than three are stable — they can be successfully stored in the network — we say that they are admissible. In order to check this hypothesis we initiate the network from several randomly chosen patterns of that type. We have not found an example of pattern which is not stable. This property will be used in the next section to estimate from below the number of patterns that can be stored in the network.

IV. NUMBER OF PATTERNS

In this section we estimate from below the number of patterns generated in the network. This computation is performed under the assumption that the trajectory of every cell can be in lower or upper half–space but three neighboring cells cannot be in the same half–space. In other words we assume that all patterns of single–cell and double–cell clusters are admissible and we do not count patterns containing clusters of more than two cells. If such patterns are also admissible we just obtain a larger number of patterns that can be generated by the network.

Let us denote by S_n the number of patterns of length n not containing clusters larger than two cells.

Let us code the ith cell in upper half–space by $\alpha_i = 1$ and the in lower half–space by $\alpha_i = 0$. We are looking for number of n-element sequences $(\alpha_1, \ldots, \alpha_n)$ with elements from the set $\{0, 1\}$ which after creating a cycle do not contain the subsequences $(1, 1, 1)$ and $(0, 0, 0)$.

Let us introduce the numbers a_n, b_n, c_n, d_n, that correspond to the number of sequences $(\alpha_i)_{i=1}^n$ with certain beginning and ending of the sequence. For example a_n is the number of n element sequences beginning with 10 and ending with 10, b_n is the number of sequences beginning with 10 and ending with 100, c_n is the number of sequences beginning with 10 and ending with 01, while d_n is the number of sequences beginning with 10 and ending with 011. One can easily prove that the numbers $a_n, \ldots, d_n$ can be computed using the following recursive formula.

$$\left\{ \begin{array}{lcl} a_{n+1} & = & c_n + d_n \\ b_{n+1} & = & a_n \\ c_{n+1} & = & a_n + b_n \\ d_{n+1} & = & c_n \end{array} \right. \tag{4}$$

with initial conditions: $a_1 = b_1 = c_1 = 0$, $d_1 = 1$.

The number of patterns S_n for $n \geq 2$ is given by

$$S_n = 2(a_n + b_n + c_n + a_{n-1} + b_{n-1}). \tag{5}$$

Using the method of generating functions we obtain the following formula for the number of steady–state patterns:

$$S_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n + 2\cos\frac{2\pi n}{3}. \tag{6}$$

The second term of S_n goes to zero as $n \to \infty$. The last term is bounded, actually it admits only two values: 2 for $n = 3k, 3k-1$ and -1 for $n = 3k+1, 3k+2$. Hence the first term decides about the behavior of S_n for large n. For large n

$$\frac{S_{n+1}}{S_n} \approx \frac{1+\sqrt{5}}{2}, \tag{7}$$

and hence the number of patterns grows 1.618 times when we increase the number of cells in the network by 1. For example for $n = 10$ cells we have 122 patterns and for $n = 30$ the number of patterns is 1860500.

Remember that different patterns correspond to different chaotic attractors. This means that the number of attractors for the system is very large.

We would like to stress that the number of patterns may be even larger due to possible stability of some patterns containing clusters of three "coherent" cells, which are not counted here.

V. CONCLUSIONS

We have performed a study of steady–state behaviors in the one–dimensional array of bi–directionally coupled chaotic circuits. We have found the range of coupling strength for which the network produces a very large number of patterns corresponding to different chaotic attractors. We have estimated the number of patterns that can be stored in the network for different network size.

ACKNOWLEDGMENTS

This research has been supported by the research fund 11.120.460 from the University of Mining and Metallurgy, Kraków, Poland.

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