Optimum Placement of Sectionalizing Switches in Distribution Networks with Alternative Supplies

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***Abstract*—The problem of optimum allocation of a given number of sectionalizing switches in distribution networks with alternative supplies to minimize a selected reliability factor is computationally hard due to a huge number of possible solutions. We present an efficient method for the computation of reliability factors in the presence of sectionalizing switches. The method uses the tree structure of radial distribution networks. Several methods to limit the search space in the optimum allocation problem are proposed. Limited search space combined with fast evaluation methods permits using the exhaustive search method for solving the allocation problem for a small number of switches and heuristic approaches requiring evaluation of the cost function for a large number of test cases. The performance of algorithms is tested using a real radial distribution network.**

I. Introduction

Reducing the frequency and duration of power interruptions to customers is one of the main objectives in the design of distribution networks [1], [2]. Improving reliability and reduction of costs associated with power outages may be achieved by using sectionalizing switches.

In this work, we study the problem of optimum placement of sectionalizing switches in radial distribution networks to improve reliability and reduce power outage costs. We will consider the problems of minimization of three reliability indexes: SAIDI (System Average Interruption Duration Index), SAIFI (System Average Interruption Frequency Index), and AENS (Average Energy Not Supplied). Several solutions to this problem have been proposed, including genetic algorithms [3], simulated annealing [4], immune algorithm [5], particle swarm optimization [6], and ant colony optimization-based method [7]. These algorithms belong to the class of heuristic approaches and can be characterized by long computation times and no guarantee that the optimum solution has been found.

In [8], a sequential optimization algorithm using thinning techniques to reduce the search space is described. A fuzzy dynamic programming approach is presented in [9], an algorithm using integer programming is described in [10], and a mixed-integer linear programming approach is presented in [11]. Improving reliability in radial distribution systems with distributed generation is studied for example in [12], [13]. Sectionalizing strategy for parallel system restoration is discussed in [14], [15]. The problem of optimal switch placement considering switch malfunction is studied in [16]. A very fast deterministic algorithm to solve the switch allocation problem in the case of networks with a single generator is described in [17].

In this work, the problem of placement of sectionalizing switches in distribution networks with multiple generators is studied. We propose a very efficient algorithm to compute reliability indexes for given positions of sectionalizing switches. The algorithm is based on the tree structure of radial distribution networks. We also study the problem how to reduce the search space. It is shown that this approach permits solving the problem using exhaustive search or heuristic methods for larger networks.

The layout of the paper is as follows. In Section II, the problem is defined, the proposed algorithms are presented in detail, and methods to limit the search space are described. In Section III, the algorithms are tested using an example distribution network and high efficiency of the algorithms is confirmed.

II. Minimization of Undelivered Energy

A. Problem definition

We assume that the distribution grid has a radial structure with m line segments and $m + 1$ nodes. The set of nodes is denoted as $V = \{v_1, v_2, \ldots, v_{m+1}\}$. For the sake of simplicity, we assume that the node v_{m+1} is the main generator (main supply node). Apart from the main supply node there are m nodes in the network: distribution nodes which are directly connected to at least two other nodes and user nodes and auxiliary generators, which are connected to a single node. The graph representation of the network has a tree structure, with the main generator being the root and load nodes and auxiliary generators being leaves (nodes without children).

Let c_j denote the connection line between the node v_j and its parent node. By λ_{v_j} and λ_{c_j} we denote the average failure rates (the average number of failures during the period of analysis; usually one year) of the node v_j and the line segment c_j, respectively. By t_{v_j} and t_{c_j} we denote the average total duration of failures during one year of the node v_j and the line segment c_j, respectively. The average total duration of failures of a given element (node or line segment) can be computed as a product of the average failure rate λ and the average failure duration τ of this element, i.e. $t_{v_j} = \lambda_{v_j}\tau_{v_j}$ and $t_{c_j} = \lambda_{c_j}\tau_{c_j}$.

Let us denote by P_j the average (active) power dissipated at the jth node and let N_j be the number of users at the jth node. We assume that for each node $P_j \geq 0$ and $N_j \geq 0$. In practice, P_j and N_j are positive for user nodes while $P_j = 0$ and $N_j = 0$ for distribution and supply nodes. *The total number of users* is $\bar{N} = \sum_{i=1}^{m} N_i$, and the *total average power* is $\bar{P} = \sum_{i=1}^{m} P_i$. The *total failure rate* is the sum of failure rates of all components (nodes and line segments) in the network $\bar{\lambda} = \sum_{i=1}^{m+1} \lambda_{v_j} + \sum_{i=1}^{m+1} \lambda_{c_j}$, and the *total interruption duration* is the sum failure durations of all components in the network $\bar{t} = \sum_{i=1}^{m+1} t_{v_j} + \sum_{i=1}^{m} t_{c_j}$.

The two most popular indexes used in reliability analysis of power networks are the System Average Interruption Frequency Index (SAIFI) and the System Average Interruption Duration Index (SAIDI) [4], [18].

SAIFI is the average number of interruptions during one year for a single user. It is defined as the total number of interruptions counted independently for each user divided by the number of users $\bar{N}$

$$\mathrm{SAIFI} = \frac{\sum_{j=1}^{m} \mu_j N_j}{\sum_{j=1}^{m} N_j}, \tag{1}$$

where m is the number of nodes, and μ_j is the outage rate of the node v_j, i.e. the average number of interruptions involving the node v_j during one year.

SAIDI is the average outage duration. It is calculated as the sum of the durations of all interruptions counted independently for each user divided by $\bar{N}$

$$\mathrm{SAIDI} = \frac{\sum_{j=1}^{m} U_j N_j}{\sum_{j=1}^{m} N_j}, \tag{2}$$

where U_j is the total duration of all interruptions involving the v_j node during one year.

The Average Energy Not Supplied (AENS) is defined as the average value of energy not supplied to users due to failures during the period of analysis

$$\mathrm{AENS} = \sum_{j=1}^{m} U_j P_j. \tag{3}$$

If there are no sectionalizing switches in the network then a failure at any location in the grid causes energy supply interruption in the entire network. In consequence, $U_j = \text{const} = \bar{t}$, $\mu_j = \text{const} = \bar{\lambda}$ and we obtain

$$\mathrm{AENS} = \bar{t} \sum_{i=1}^{m} P_i = \bar{t} \cdot \bar{P}, \quad \mathrm{SAIFI} = \bar{\lambda}, \quad \mathrm{SAIDI} = \bar{t}. \tag{4}$$

For a given network, coefficients SAIFI, SAIDI, and AENS can be reduced by introducing sectionalizing switches at selected line segments. In case of a failure, we may disconnect a part of the grid and energy supply to the remaining part of the grid may be continued in spite of the fault.

Let $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_P\}$ denotes the set of admissible positions of sectionalizing switches. Switches may be placed at both ends of each line segment and hence the maximum number of switch positions is $P = 2m$. However, as will be discussed later, some switch positions may be excluded, so in a general case $P \leq 2m$.

Let $Q = \{\gamma_{i_1}, \gamma_{i_2}, \ldots, \gamma_{i_p}\} \subset \Gamma$ be a selected set of positions of p sectionalizing switches. Let us denote by AENS(Q) the average energy not supplied if there are switches at the positions belonging to Q. Similarly, we define SAIFI(Q) and SAIDI(Q) to be the SAIFI and SAIDI indexes for the case of sectionalizing switches in Q.

The optimization problem to be solved is to find for a given $p \in \{1, 2, \ldots, P\}$ the minimum value of AENS which can be obtained using p sectionalizing switches

$$\mathrm{AENS}_{\min}(p) = \min_{Q:\, \#Q = p} \mathrm{AENS}(Q), \tag{5}$$

and the corresponding positions of switches, where $\#Q$ denotes the carnality of Q.

The optimization problems involving SAIDI and SAIFI indexes are find positions of p sectionalizing switches which minimize SAIDI(Q) and SAIFI(Q) under the condition $\#Q = p$.

B. *Efficient computation of* SAIFI, SAIDI, *and* AENS

In this section, we present a fast algorithm for the evaluation of reliability indexes when positions of sectionalizing switches are fixed. In the first step, we construct a tree structure of admissible positions of sectionalizing switches. The set of vertices in this structure is $\{\gamma_0\} \cup \Gamma$, where γ_0 denotes the supply node and Γ is the set of admissible positions of sectionalizing switches. The vertex γ_0 is the root of the tree structure. There is an edge between two vertices in $\{\gamma_0\} \cup \Gamma$ if there is a direct connection between the corresponding positions in the distribution network not passing through another admissible position. Since there is a single path from any admissible position to the generator γ_0 it follows that besides γ_0 each vertex has a single parent. For a given vertex γ_j by C_j we denote the set of children of this vertex (i.e. the set of vertices whose parent is γ_j) and by D_j the set of descendants of γ_j. For each vertex γ_j we define quantities $\bar{P}_j$, $\bar{N}_j$, $\bar{t}_j$, and $\bar{\lambda}_j$. $\bar{P}_j$ and $\bar{N}_j$ are the total average power and the total number of users for nodes being descendants of γ_j. $\bar{t}_j$ and $\bar{\lambda}_j$ are the sum of average failure times and the sum of failure rates of network elements being descendants of γ_j.

We say that a position γ_k is *single powered* if there is a generator only on one side of this position. Otherwise, we say that the position γ_k is *double-powered*. Similarly, we define single- and double-powered line segments.

First, we present the tree-structure based algorithm for the computation of AENS. Let us consider an arbitrary set $Q \subset \Gamma$. Let us assume that the set Q has p elements. Placing p switches in a network splits the network into $p+1$ components. Each $\gamma_k \in Q$ starts a single component. The last component starts at the generator γ_0. Let us consider the kth component starting at a position $\gamma_k \in \{\gamma_0\} \cup Q$. Let us denote by R_k the set of switches $\gamma_j \in D_j$ which can be reached from γ_k without passing through another switch in Q. With this definition the total failure time of network elements belonging to this component can be computed as $\left(\bar{t}_k - \sum_{\gamma_j \in R_k} \bar{t}_j\right)$. Let us denote by T_k the subset of switches in R_k which are double-powered. Users behind switches in T_k are not affected by failures in the kth component. Hence, the average total power of users affected by failures of element in the kth component is $\left(\bar{P}_k - \sum_{\gamma_j \in T_k} \bar{P}_j\right)$. It follows that the average energy not supplied if there are switches in Q can be computed as

$$\text{AENS}(Q) = \sum_{\gamma_k \in \{\gamma_0\} \cup Q} \left(\bar{P}_k - \sum_{\gamma_j \in T_k} \bar{P}_j\right)\left(\bar{t}_k - \sum_{\gamma_j \in R_k} \bar{t}_j\right). \tag{6}$$

For the first component γ_0 we have $\bar{P}_0 = \bar{P}$ and $\bar{t}_k = \bar{t}$. After some algebraic manipulation the equation (6) can be rewritten as

$$\begin{aligned}\text{AENS}(Q) = &\bar{P} \cdot \bar{t} - \left(\sum_{\gamma_j \in T_0} \bar{P}_j\right)\left(\bar{t} - \sum_{\gamma_j \in R_0} \bar{t}_j\right) \\ &- \sum_{\gamma_k \in Q} \left(\bar{P} - \bar{P}_k + \sum_{\gamma_j \in T_k} \bar{P}_j\right)\left(\bar{t}_k - \sum_{\gamma_j \in R_k} \bar{t}_j\right).\end{aligned} \tag{7}$$

The first component $\bar{P} \cdot \bar{t}$ in (7) is the average energy not supplied in case of no sectionalizing switches and the remaining part is the gain obtained by using switches in Q.

In case of a network without auxiliary generators $T_k = \emptyset$ for each γ_k and in consequence (7) reduces to

$$\text{AENS}(Q) = \bar{P} \cdot \bar{t} - \sum_{\gamma_k \in Q} \left(\bar{P} - \bar{P}_k\right)\left(\bar{t}_k - \sum_{\gamma_j \in R_k} \bar{t}_j\right), \tag{8}$$

A tree structure based method to compute AENS(Q) is presented as the Algorithm 1.

Algorithm 1 Tree algorithm to compute AENS(Q).

Precondition: Q is the set of switch positions

1: **function** VISITNODE(Q, γ_ℓ)
2: $\quad (g_\ell, a_\ell, x_\ell) \leftarrow (0, 0, 0)$
3: $\quad$ **for** $\gamma_i \in C_\ell$ **do**
4: $\quad\quad (g_i, a_i, x_i) \leftarrow$ VISITNODE(Q, γ_i)
5: $\quad\quad (g_\ell, a_\ell, x_\ell) \leftarrow (g_\ell + g_i, a_\ell + a_i, x_\ell + x_i)$
6: $\quad$ **end for**
7: $\quad$ **if** $\gamma_\ell \in \{\gamma_0\}$ **then**
8: $\quad\quad g_\ell \leftarrow g_\ell + x_\ell(\bar{t}_k - a_\ell)$
9: $\quad$ **end if**
10: $\quad$ **if** $\gamma_\ell \in Q$ **then**
11: $\quad\quad g_\ell \leftarrow g_\ell + (\bar{P} - \bar{P}_k + x_\ell)(\bar{t}_k - a_\ell)$
12: $\quad\quad a_\ell \leftarrow \bar{t}_k$
13: $\quad\quad$ **if** γ_ℓ is double-powered **then**
14: $\quad\quad\quad x_\ell \leftarrow \bar{P}_k$
15: $\quad\quad$ **end if**
16: $\quad$ **end if**
17: $\quad$ **return** (g_ℓ, a_ℓ, x_ℓ)
18: **end function**
19: **function** AENS(Q)
20: $\quad (g_0, a_0, x_0) \leftarrow$ VISITNODE(Q, γ_0)
21: $\quad$ **return** $\bar{P} \cdot \bar{t} - g_0$
22: **end function**

To explain the algorithm let us introduce the notion of a partial solution. The *partial solution* at the position $\gamma_\ell \in \{\gamma_0\} \cup \Gamma$ involves switches located in the set D_ℓ. $Q_\ell = Q \cap D_\ell$ is the set of switches in the partial solution. For the partial solution at the position γ_ℓ, we define the *gain* g_ℓ

$$g_\ell = \sum_{\gamma_k \in Q_\ell} \left(\bar{P} - \bar{P}_k + \sum_{\gamma_j \in T_k} \bar{P}_j\right)\left(\bar{t}_k - \sum_{\gamma_j \in R_k} \bar{t}_j\right). \tag{9}$$

In the Algorithm 1 the sums $\sum_{\gamma_j \in T_k} \bar{P}_j$ and $\sum_{\gamma_j \in R_k} \bar{t}_j$ are represented as x_k and a_k, respectively. Note that x_k becomes non-zero only at switches γ_k which are double-powered.

To compute AENS(Q) we visit the tree $\{\gamma_0\} \cup \Gamma$ using the depth-first search (DFS) algorithm [19]. The algorithm is started at the root vertex γ_0. In the DFS algorithm computations for a given vertex are carried out after all its children have been processed. The Algorithm 1 recursively computes gains g_k and sums a_k, and x_k for partial solutions starting from leaf nodes and moving towards the root node γ_0. At the final step the gain at the root node is subtracted from AENS($\emptyset$) = $\bar{P} \cdot \bar{t}$. The computations can be done in a single pass of the tree structure, and in consequence the computations are very fast.

SAIDI and SAIFI indexes can be computed using the following formulas

$$\begin{aligned}\text{SAIDI}(Q) = &\bar{t} - \frac{1}{\bar{N}}\left(\sum_{\gamma_j \in T_0} \bar{N}_j\right)\left(\bar{t} - \sum_{\gamma_j \in R_0} \bar{t}_j\right) \\ &- \frac{1}{\bar{N}} \sum_{\gamma_k \in Q} \left(\bar{N} - \bar{N}_k + \sum_{\gamma_j \in T_k} \bar{N}_j\right)\left(\bar{t}_k - \sum_{\gamma_j \in R_k} \bar{t}_j\right),\end{aligned} \tag{10}$$

$$\begin{aligned}\text{SAIFI}(Q) = &\bar{\lambda} - \frac{1}{\bar{N}}\left(\sum_{\gamma_j \in T_0} \bar{N}_j\right)\left(\bar{\lambda} - \sum_{\gamma_j \in R_0} \bar{\lambda}_j\right) \\ &- \frac{1}{\bar{N}} \sum_{\gamma_k \in Q} \left(\bar{N} - \bar{N}_k + \sum_{\gamma_j \in T_k} \bar{N}_j\right)\left(\bar{\lambda}_k - \sum_{\gamma_j \in R_k} \bar{\lambda}_j\right).\end{aligned} \tag{11}$$

The algorithms to evaluate SAIDI(Q) and SAIFI(Q) are very similar to the Algorithm 1. Changes in the Algorithm 1 are as follows. To compute SAIDI(Q) we have to replace $\bar{P}_j$ by $\bar{N}_j$, $\bar{P}$ by $\bar{N}$ and replace $\bar{P} \cdot \bar{t} - g_0$ in line 21 by $\bar{t} - g_0/\bar{N}$. To compute SAIFI(Q) we have to replace $\bar{P}_j$ by $\bar{N}_j$, $\bar{P}$ by $\bar{N}$, $\bar{t}_j$ by $\bar{\lambda}_j$ and replace $\bar{P} \cdot \bar{t} - g_s$ by $\bar{\lambda} - g_0/\bar{N}$ in the line 21.

C. *Limiting the search space*

As mentioned before for a network with m line segments the number of admissible switch positions is $P \leq 2m$. For $P = 2m$ the exhaustive search approach in which all possible selections of p switches are considered to find the minimum value of a given reliability factor requires evaluation of this factor for $N_{\text{ES}} = \binom{2m}{p}$ of test selections. N_{ES} grows very fast with p for large m. A large number of test selections also makes the problem harder for heuristic approaches which require dense sampling of the search space. Therefore, in order to achieve a good performance of optimization algorithms it is essential to limit the search space.

First, let us note that we do not need to consider sectionalizing switches at positions closest to each supply node. The line segment at the main generator always contains a switch which is normally closed, while line segments at alternative generators contain switches which are normally opened.

Second, in case of single-powered line segments it is better to place a switch at the end which is closer to the power supply. This choice guarantees that this switch can be activated for all failures involving this line segment.

Using these two observations, the number of admissible positions for a network with m line segments, m_g generators and m_s single-powered line segments can be reduced to $P = 2m - m_g - m_s$.

An example is shown in Fig. 1. The network contains $m = 11$ line segments. Supply nodes, distribution nodes, and user nodes are plotted as red squares, yellow circles, and green hexagons, respectively. The main generator is at the node 12 and the auxiliary generator is at the node 1. The circuit breaker at the node 12 plotted as a solid black circle is normally closed and the circuit breaker at the auxiliary generator is normally open. Admissible positions of sectionalizing switches are plotted as thick short intervals perpendicular to line segments. There are $m_g = 2$ generators and $m_s = 7$ single-powered line segments. Hence, the number of admissible positions is $P = 2m - m_g - m_s = 13$.

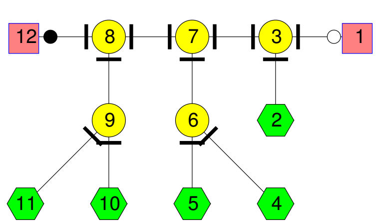

Fig. 1. A example distribution grid with two generators and $m = 11$ line segments. Admissible positions of sectionalizing switches are plotted as thick short intervals. The circuit breaker at the node 12 is normally closed. The circuit breaker at the node 1 is normally open.

The number of selections can be further reduced by eliminating partial solutions which cannot lead to optimal solutions. This can be done for partial solutions involving single-powered parts of the network using the algorithm presented in [17]. In this approach first all double-powered line segments are identified. The remaining part of the graph representing the network is a union of single-powered connected components. For the network shown in Fig. 1 there are four double-powered line segments. They belong to the path connecting two generators (nodes 1 and 12). There are three single-powered components. One of them contains a single line segment (the edge starting at the node number 2). The other two contain three line segments each (edges starting at the nodes 4, 5, 6 and edges starting at the nodes 9, 10, 11). Let $p_{\max}$ be the maximum number of sectionalizing switches to be considered. Using the algorithm presented in [17], for each single-powered component we compute the set of partial solutions containing up to $p_{\max}$ switches and we skip those which cannot lead to the optimum solution. The remaining set of partial solutions is stored. The search space is the set of complete solutions with no more than $p_{\max}$ switches being combinations of admissible positions of switches at double-powered line segments and partial solutions found in the previous step. In the next section, we show that this method significantly reduces the number of complete solutions. In consequence, the exhaustive search method can be carried out for larger $p_{\max}$. With the reduced search space heuristic algorithms require fewer test solutions to find acceptable positions of sectionalizing switches.

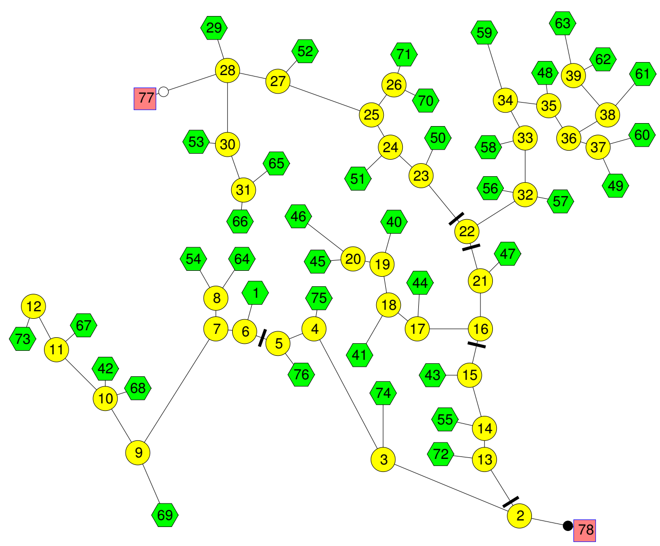

Fig. 2. A distribution grid with two generators and $m = 77$ line segments. The circuit breaker at the node 78 is normally closed. The circuit breaker at the node 77 is normally open. Optimum positions of $p = 5$ sectionalizing switches for the minimization of AENS are denoted with short thick lines intersecting connection lines.

III. Computational Example

As an example, let us consider a network with $m = 77$ line segments shown in Fig. 2. Data for this network is provided by an electricity company located in the southern part of Poland. The data includes topology of the network, lengths and types of line segments, average power and the number of users for load nodes, and history of faults. Based on the analysis of failures occurring during a two years period, failure rates and average failure durations for different types of elements have been computed. The average number of faults is 3.1 in one year for every 100 km of line segment. The average failure rate of a given line segment c_j with the length in meters equal to l_j can be computed as $\lambda_{c_j} = 3.1 \times 10^{-5} l_j$. The average fault duration is $\tau_{c_j} = 0.983\,\mathrm{h}$. The average total duration of failures of the line segment c_j is $t_{c_j} = \tau_{c_j} \lambda_{c_j} = 0.983 \cdot 3.1 \times 10^{-5} l_j$. The average number of faults in one year is $\lambda_{v_j} = 0.03$ for user nodes and $\lambda_{v_j} = 0.002$ for distribution nodes. The average duration of the fault is $\tau_{v_j} = 1\,\mathrm{h}$ for user nodes and $\tau_{v_j} = 0.5\,\mathrm{h}$ for distribution nodes.

First, let us test the speed of the algorithm for evaluation of the average energy not supplied. We have carried out the

TABLE I

OPTIMIZATION OF AENS USING THE EXHAUSTIVE SEARCH APPROACH WITH VARIOUS METHODS TO LIMIT THE SEARCH SPACE; N_{ES} DENOTES THE NUMBER OF EVALUATIONS; t_{ES} DENOTES THE TOTAL COMPUTATION TIME.

p	N_{ES}	t_{ES} [s]	N_{ES2}	t_{ES2} [s]	N_{ES3}	t_{ES3} [s]	N_{ES4}	t_{ES4} [s]
0	1	0.00	1	0.00	1	0.00	1	0.00
1	154	0.00	88	0.00	47	0.00	38	0.00
2	11781	0.14	3828	0.06	1081	0.02	697	0.02
3	$5.97 \cdot 10^5$	3.77	$1.10 \cdot 10^5$	0.74	16234	0.25	8235	0.14
4	$2.25 \cdot 10^7$	142.93	$2.33 \cdot 10^6$	14.65	$1.79 \cdot 10^5$	2.01	70555	0.71
5	$6.76 \cdot 10^8$	4423.42	$3.92 \cdot 10^7$	249.80	$1.55 \cdot 10^6$	16.92	$4.67 \cdot 10^5$	4.56
6			$5.41 \cdot 10^8$	3586.63	$1.10 \cdot 10^7$	128.34	$2.50 \cdot 10^6$	23.99
7			$6.35 \cdot 10^9$	41231.31	$6.54 \cdot 10^7$	781.40	$1.11 \cdot 10^7$	107.66
8					$3.35 \cdot 10^8$	3542.02	$4.18 \cdot 10^7$	401.89
9					$1.50 \cdot 10^9$	16838.91	$1.36 \cdot 10^8$	1337.15
10							$3.83 \cdot 10^8$	3884.76
11							$9.56 \cdot 10^8$	9665.40
12							$2.12 \cdot 10^9$	22301.29

exhaustive search in which all possible selections of p switches are considered to find the minimum value of AENS for a given p. The undelivered energy is computed using the fast evaluation method presented in Section II. The results are presented in Table I. We report the number $N_{\mathrm{ES}} = \binom{2m}{p} = \binom{154}{p}$ of evaluations in the exhaustive search method, and the computation time t_{ES}. Computations have been carried out using a single core 3.1 GHz processor. One can see that the computation algorithm can handle approximately 150000 selections in one second. Due to a very large search space we are able to solve the problem using the exhaustive search approach for $p \leq 5$.

For larger p, the problem can be solved using the exhaustive search by reducing the number of admissible positions of switches. The test network contains 13 double-powered line segments, $m_s = 64$ single-powered line segments and $m_g = 2$ supply nodes. Hence, the number of admissible positions can be reduced to $P = 2m - m_g - m_s = 88$. In this version (ES2) the number of evaluations is $N_{\mathrm{ES2}} = \binom{P}{p} = \binom{88}{p}$. The computation time t_{ES2} is reported in Table I. The number of tested selections and the computation time for $p = 5$ are reduced approximately 17 times when compared with the first version.

Further reduction can be obtained using the third version (ES3) in which non-optimum partial solutions are skipped as explained in the previous section. Using this version the problem has been solved for $p \leq 9$. The number of selections which have to be considered depends of the number of stored partial selections for each single-powered component. The number of selections for $p = 7$ is reduced approximately 100 times when compared to the second version.

In the last version (ES4), sectionalizing switches in line segments starting at user nodes are not considered. Using this version permits finding optimum solutions for $p \leq 12$. However, one should remember that this version may produce suboptimal results, especially when the number of switches is high.

From the results presented above, it follows that the evaluation algorithms are very efficient and the search space is considerably reduced by using the proposed methods. We conclude with the statement that the proposed methods can be effectively used for solving switch allocation problems using exhaustive search for a small number of switches. The algorithm may also be useful in heuristic methods where reliability indexes has to be evaluated for many test selections.

Let us now study what is the influence of having auxiliary generators on reliability indexes for the network presented in Fig. 2. We consider three cases. In the first case ($m_g = 1$) there is only a single generator at the node 78. In the second case ($m_g = 2$) there is additionally an auxiliary generator at the node 77 (this case has been considered so far). In the third case ($m_g = 3$) there are two auxiliary generators at nodes 69 and 77.

TABLE II

COMPARISON OF RELIABILITY INDEXES FOR NETWORKS WITH ONE, TWO, AND THREE SUPPLY NODES. RESULTS ARE GIVEN RELATIVE TO THEIR VALUES FOR $p = 0$: AENS($\emptyset$) = 7159, SAIDI($\emptyset$) = 2.329.

	one supply node		two supply nodes		three supply nodes	
p	AENS	SAIDI	AENS	SAIDI	AENS	SAIDI
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.6619	0.7371	0.4970	0.4998	0.4970	0.4998
2	0.5049	0.5424	0.3477	0.3516	0.3359	0.3516
3	0.4183	0.4288	0.2555	0.2513	0.2420	0.2508
4	0.3397	0.3247	0.2238	0.2225	0.1967	0.2039
5	0.2891	0.2906	0.2011	0.1974	0.1673	0.1789
6	0.2574	0.2725	0.1806	0.1793	0.1468	0.1571
7	0.2430	0.2564	0.1661	0.1633		
8	0.2316	0.2420	0.1563	0.1536		

Optimum values of reliability indexes AENS, SAIDI for different number of sectionalizing switches which are obtained in these three cases are presented in Table II. The indexes are reported relative to their values for the case of no sectionalizing switches ($p = 0$). The results for the SAIFI indexes are similar to those obtained for the SAIDI index and are not reported.

Installation of a single switch ($p = 1$) decreases AENS by 50% for the network with two supply nodes ($m_g = 2$) and only by 34% for the network with a single supply node ($m_g = 1$). Even larger difference is observed for the SAIDI index. In this case ($p = 1$) using three generators ($m_g = 3$) does not change the results when compared with the case of two generators. This is due to the fact with a single sectionalizing switch ($p = 1$) one cannot divide the networks into three parts and two generators always handle the same set of load nodes. However, already with $p = 2$ reliability indexes are improved by using three generators. One can see that the improvement grows with p.

It is interesting to note that the minimum relative value of AENS which can be obtained for the network with a single generator is 0.1651. This is achieved by installing switches in all 76 admissible positions. On the other hand for the network with two supply nodes the relative value of AENS can be reduced to 0.1563 using just 8 switches. Similar phenomena are observed for other indexes. It follows that using auxiliary generators might be necessary to decrease reliability indexes to a given value.

In Table III we report the optimum values of the SAIDI index which can be obtained by installing p sectionalizing switches. These values are compared with the SAIDI index obtained for positions of sectionalizing switches minimizing AENS. Similarly, we report the optimum values of AENS and values of AENS for positions of sectionalizing switches optimizing SAIDI. One can see that in several cases the compared values differ significantly. Optimizing a given factor may not necessarily lead to a close-to-optimum values of other indexes. It follows that if the goal is to simultaneously optimize several reliability measures then a better approach may be to use a multi-objective optimization. This problem is left for future study.

TABLE III
OPTIMUM VALUE OF THE SAIDI INDEX VERSUS SAIDI FOR SECTIONALIZING SWITCHES OPTIMIZING AENS.

p	$\mathrm{SAIDI}_{\mathrm{OPT}}$	$\mathrm{SAIDI}_{\mathrm{AENS}}$	$\mathrm{AENS}_{\mathrm{OPT}}$	$\mathrm{AENS}_{\mathrm{SAIDI}}$
0	1.0000	1.0000	1.0000	1.0000
1	0.4998	0.5385	0.4970	0.4972
2	0.3516	0.3520	0.3477	0.3514
3	0.2513	0.2514	0.2555	0.2558
4	0.2225	0.2359	0.2238	0.2328
5	0.1974	0.2070	0.2011	0.2123
6	0.1793	0.1819	0.1806	0.1825
7	0.1633	0.1658	0.1661	0.1680
8	0.1536	0.1536	0.1563	0.1563
9	0.1448	0.1448	0.1493	0.1493

IV. CONCLUSIONS

Efficient algorithms for the evaluation of reliability indexes for radial distribution networks with alternative supplies in the presence of sectionalizing switches have been presented. Methods to reduce the search space in the problem of optimum allocation of switches have been described. The proposed approach permits solving switch allocation problems using the exhaustive search method for a small number of switches and heuristic methods which require handling large number of test selections. Algorithms have been tested using a real network of a moderate size.

ACKNOWLEDGMENT

This work was supported by the National Science Centre, Poland, grant no. 2014/15/B/ST8/02315.

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