Return Map Approach for Simulations of Electronic Circuits with Memristors

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Plan of the talk

- Memristors.
- Memristor models:
  - Strukov model.
  - Joglekar window.
  - Biolek window.
  - VTeam model.
- Simulations of electronic circuits with memristors:
  - discontinuities in memristor models,
  - return map approach for simulations of memristors.
- Simulation example:
  - a sinusoidally driven memristor described using the VTEAM model,
  - a comparison with standard numerical integration methods.
- Conclusions.

Memristors

- A classical memristor is a circuit element characterized by a nonlinear relation between charge and flux. Its resistance depends on the history of current flowing through the element or the voltage across the element.
- Memristors were theoretically introduced in 1971.
- Memristive behavior was recognized in a nano-scale double-layer $\mathrm{TiO_2}$ film in 2008.
- Memristors have received significant attention due to a number of possible applications including large-capacity nonvolatile memories and neuromorphic systems.

Current-controlled and voltage-controlled memristors

- A current-controlled memristor:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x, i),$$
$$v(t) = R(x, i)i(t),$$

  where x is the internal variable of the memristor, $R(x, i)$ is called the memristance, $v(t)$ is the voltage across the device, and $i(t)$ is the current passing through the device.

- A voltage-controlled memristor:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = h(x, v),$$
$$i(t) = G(x, v)v(t),$$

  where $G(x, v)$ is called the memductance.

Modeling of memristors

- In several models, the internal state of a memristor is limited by physical dimensions of the device.
- Limitation of variables introduce discontinuities in right hand sides (RHS) of differential equations describing dynamics of circuits with memristors.
- These discontinuities make it difficult to integrate such systems using standard numerical integration methods.
- An approach based on the concept of a return map is proposed to solve this problem.
- A simple example is discussed to show the usefulness of the proposed technique.

The linear ion drift model (Strukov model)

- Physical model: two-layer thin film (width $D \approx 10\ \mathrm{nm}$),
- One layer (of width $w \in [0, D]$) is doped with oxygen vacancies (behaves as a semiconductor).
- The second layer (of width $D - w$) behaves as an insulator.
- The internal variable $x = w/D \in [0, 1]$.
- The linear ion drift model

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ki(t)f(x),$$
$$v(t) = R(x)i(t), \quad R(x) = R_{\mathrm{on}}x + R_{\mathrm{off}}(1 - x).$$

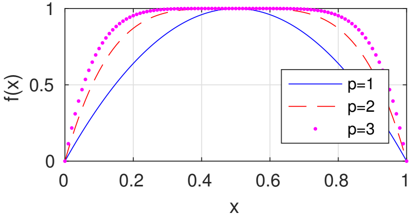
- The ideal rectangular window function

$$f(x) = \begin{cases} 1, \ x \in [0, 1], \\ 0, \ \text{otherwise}. \end{cases}$$

- Modeling problem: the RHS of the equation defining $\mathrm{d}w/\mathrm{d}t$ is discontinuous, standard integration methods may work improperly for ODEs with discontinuous right hand sides.

Joglekar window

- The idea: decrease the rate of change of x close to the window bounds and make the window function continuous.
- The Joglekar window function:
  $f(x) = 1 - (2x - 1)^{2p}$ for $x \in [0, 1]$, $f(x) = 0$, for $x \notin [0, 1]$:



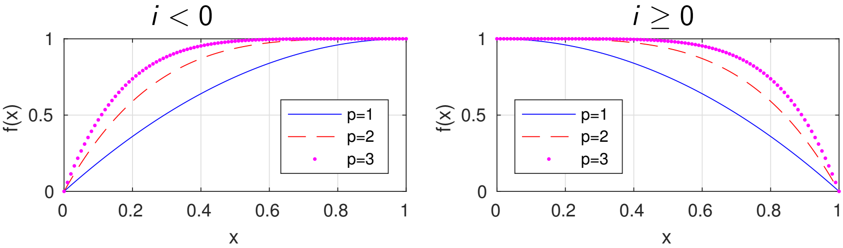
- The Joglekar window function is continuous.
- Modeling problem: when w reaches a boundary, the right hand side of the equation defining $\mathrm{d}w/\mathrm{d}t$ is zero and the internal variable may remain constant for ever.

Biolek window

- The Biolek window function:

$$f(x,i) = \begin{cases} 1 - (x - \mathrm{stp}(-i))^{2p}, & w > 0, i < 0 \text{ or } w < 1, i > 0, \\ 0, & \text{otherwise.} \end{cases}$$

where $\mathrm{stp}(i) = 0$ for $i < 0$ and $\mathrm{stp}(i) = 1$ for $i \geq 0$.



- An artificial construction to avoid modeling problems at boundaries.

VTEAM model

- The VTEAM model introduces threshold voltages v_{on} and v_{off}

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \begin{cases} k_{\text{off}}\left(v/v_{\text{off}} - 1\right)^{\alpha_{\text{off}}} f_{\text{off}}(w), & 0 < v_{\text{off}} < v, \\ 0, & v_{\text{on}} \le v \le v_{\text{off}}, \\ k_{\text{on}}\left(v/v_{\text{on}} - 1\right)^{\alpha_{\text{on}}} f_{\text{on}}(w), & v < v_{\text{on}} < 0, \end{cases}$$

- The resistance remains constant for $v_{\text{on}} \le v \le v_{\text{off}}$.
- $f_{\text{off}}(w)$, and $f_{\text{on}}(w)$ are window functions which constrain the internal variable w to bounds $[w_{\text{on}}, w_{\text{off}}]$.
- The ideal rectangular window function:

$$f_{\text{off}}(w) = f_{\text{on}}(w) = \begin{cases} 1, & w \in [w_{\text{on}}, w_{\text{off}}], \\ 0, & \text{otherwise}. \end{cases}$$

- A linear dependence between $R(w)$ and w:

$$i(t) = \left(R_{\text{on}} + \frac{w - w_{\text{on}}}{w_{\text{off}} - w_{\text{on}}}(R_{\text{off}} - R_{\text{on}})\right)^{-1} u(t).$$

Return map approach

- The state space $\mathbb{R}^n$ is divided into m smooth regions $\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_m$ with boundaries $\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_m$. For the region $\mathcal{R}_k$, we define the return map P_k with the return condition defined using the boundary $\mathcal{B}_k$.
- For $x \in \mathbb{R}^n$ the image $P_k(x)$ under the return map P_k is defined as the first intersection of the trajectory starting at x with $\mathcal{B}_k$.
- The return map approach,
    - a standard numerical integration method is used to compute the trajectory as long as it remains in $\mathcal{R}_k$,
    - during the evaluation of P_k the return condition defined by $\mathcal{B}_k$ is monitored,
    - after entering another smooth region computations are continued for the corresponding return map.
- The main advantage: we never numerically integrate a discontinuous or a non-smooth system.

Return map approach example: the VTEAM model

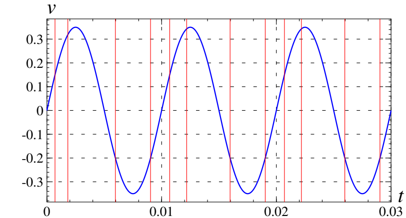
- Five smooth regions separated by conditions: $w = w_{\rm off}$, $w = w_{\rm on}$, $v = v_{\rm off}$, and $v = v_{\rm on}$:
  - Region $\mathcal{R}_1$: $v > v_{\rm off}$, $w < w_{\rm off}$; the variable w grows according to $\dot{w} = k_{\rm off}\,(v/v_{\rm off} - 1)^{\alpha_{\rm off}}$; the return map condition: $(v - v_{\rm off})(w - w_{\rm off}) = 0$; if $v = v_{\rm off}$ go to region $\mathcal{R}_3$, if $w = w_{\rm off}$ go to region $\mathcal{R}_4$.
  - Region $\mathcal{R}_2$: $v < v_{\rm on}$, $w > w_{\rm on}$, the variable w decreases according to $\dot{w} = k_{\rm on}\,(v/v_{\rm on} - 1)^{\alpha_{\rm on}}$, the return map condition: $(v - v_{\rm on})(w - w_{\rm on}) = 0$; if $v = v_{\rm on}$ go to region $\mathcal{R}_3$, if $w = w_{\rm on}$ go to region $\mathcal{R}_5$.
  - Region $\mathcal{R}_3$: $v \in (v_{\rm on}, v_{\rm off})$, $w \in (w_{\rm on}, w_{\rm off})$; $\dot{w} = 0$; the return map condition: $(v - v_{\rm off})(v - v_{\rm on}) = 0$; if $v = v_{\rm off}$ go to region $\mathcal{R}_1$, if $v = v_{\rm on}$ go to region $\mathcal{R}_2$,
  - Region $\mathcal{R}_4$: $v > v_{\rm on}$, $w = w_{\rm off}$; $\dot{w} = 0$; the return map condition $v - v_{\rm on} = 0$; if $v = v_{\rm on}$ go to Region $\mathcal{R}_2$.
  - Region $\mathcal{R}_5$: $v < v_{\rm off}$, $w = w_{\rm on}$; $\dot{w} = 0$, the return map condition: $v - v_{\rm off} = 0$; if $v = v_{\rm off}$ go to Region $\mathcal{R}_1$.

Simulation example: sinusoidally driven memristor

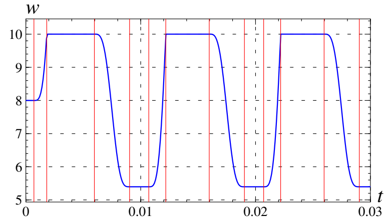
- VTEAM model: $\alpha_{\mathrm{off}} = \alpha_{\mathrm{on}} = 3$, $v_{\mathrm{off}} = 0.15$ V, $v_{\mathrm{on}} = -0.20$ V, $R_{\mathrm{off}} = 1000\ \Omega$, $R_{\mathrm{on}} = 100\ \Omega$, $w_{\mathrm{on}} = 0$, $w_{\mathrm{off}} = 10$ nm, $k_{\mathrm{off}} = 4 \cdot 10^{-6}$ m/s, $k_{\mathrm{on}} = -8 \cdot 10^{-6}$ m/s.
- The input voltage: $v(t) = V_m \sin(\omega t)$, where $V_m = 0.4$ V, $\omega = 2\pi/T$, $T = 0.01$ s.
- Explicit solutions in each smooth region:
  - Region $\mathcal{R}_1$: $v > v_{\mathrm{off}}$ and $w < w_{\mathrm{off}}$;
$$w(t) = w(t_0) + \int_{t_0}^{t} k_{\mathrm{off}} \left(\frac{v(t)}{v_{\mathrm{off}}} - 1 \right)^{\alpha_{\mathrm{off}}} \mathrm{d}t.$$
  - Region $\mathcal{R}_2$: $v < v_{\mathrm{on}}$ and $w > w_{\mathrm{on}}$;
$$w(t) = w(t_0) + \int_{t_0}^{t} k_{\mathrm{on}} \left(\frac{v(t)}{v_{\mathrm{on}}} - 1 \right)^{\alpha_{\mathrm{on}}} \mathrm{d}t.$$
  - Other regions: $w(t) = \mathrm{const.}$
- For $\alpha_{\mathrm{off}} = \alpha_{\mathrm{on}} = 3$ solutions in regions 1 and 2 can be found using an explicit formula for the indefinite integral.

Simulation example: return map approach

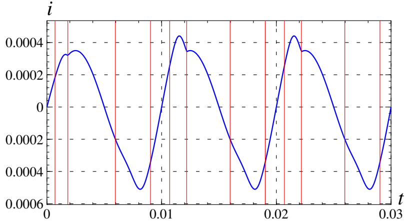
the input signal [V]



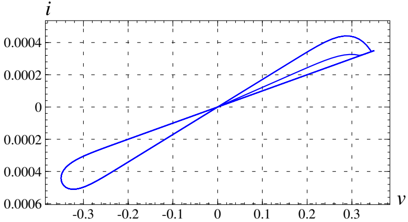
the internal variable w [nm]



the current [A]



the v-i plot

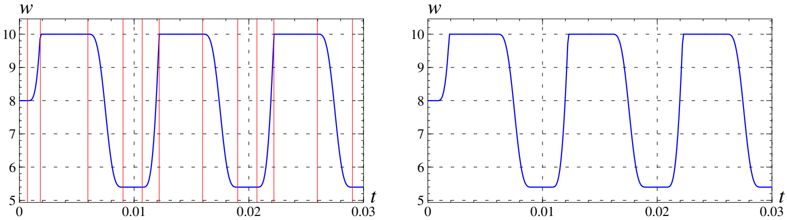


- Red vertical lines: changes of a smooth regions $\mathcal{R}_k$.

Simulation example: return map approach

- In general, analytical solutions are not available.
- Example: solution obtained using the first order Euler method with the fixed time step $\tau = 10^{-4}$.

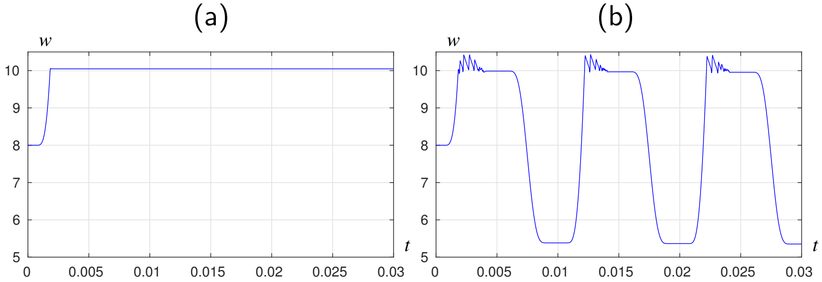
explicit formulas — the Euler method



- The results are very close to the ones obtained using the explicit formulas.

Simulation example: standard numerical methods

- The `ode45` procedure from the MATLAB package
  - (a) with the RHS being zero outside the interval $[w_{\mathrm{on}}, w_{\mathrm{off}}]$: the trajectory reaches the region $w > w_{\mathrm{off}}$ (numerical errors) and stays there for ever,
  - (b) with the RHS forcing solutions to converge to $[w_{\mathrm{on}}, w_{\mathrm{off}}]$: high frequency oscillations at the boundary w_{off}.



Conclusions

- A return map based approach for simulations of circuits containing memristor elements has been introduced.
- Its usefulness has been shown using a sinusoidally driven memristor described by the VTEAM model.