Akademia Górniczo-Hutnicza
im. Stanisława Staszica w Krakowie
AGH University of Science
and Technology

Modeling Sinusoidally Driven Self-directed Channel Memristors

Bartłomiej Garda and Zbigniew Galias

AGH University of Science and Technology
Department of Electrical Engineering
al. Mickiewicza 30, 30-059 Kraków, Poland

ICSES, Kraków 2018
International Conference on Signals and Electronic Systems

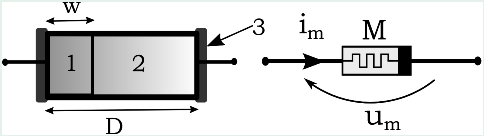
Problem description

Abstract

- In the presentation, the problem of memristors modeling is investigated.
- The elements under study are SDC (self-directed-channel) memristors with a tungsten dopant fabricated by the Knowm Inc.
- Three existing memristor models are considered: the Strukov model, the Biolek model, and the VTEAM model.
- Parameters of the models are fitted to experimental data using the interior-point optimization algorithm.
- Based on the results obtained comparison of models are going to be presented.

Problem description

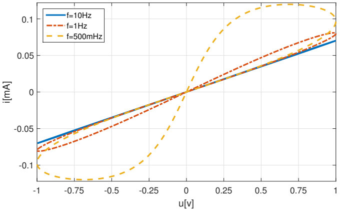
Memristor is the famous fourth basic electric circuit element postulated by Leon Chua in 1971. The discovery of the physical solid-state structures having memristive characteristics by the HP scientists in 2008 has attracted enormous interest in the scientific community.

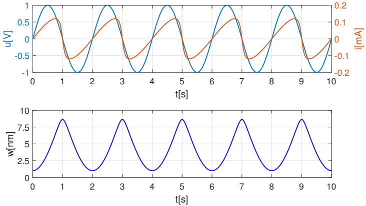


Model of the memristor and its equivalent symbol.
(1-doped region, 2-undoped region, 3-metal contact)

Memristor sinusoidal response

a) - The typical zero-crossing pinched hysteresis loop as the memristor response for the sinusoidal excitation ($U_{\mathrm{max}} = 1\mathrm{V}$).
b) - The voltage and the current in the memristor and the $w(t)$ function vs time t for $f = 0.5\mathrm{Hz}$.

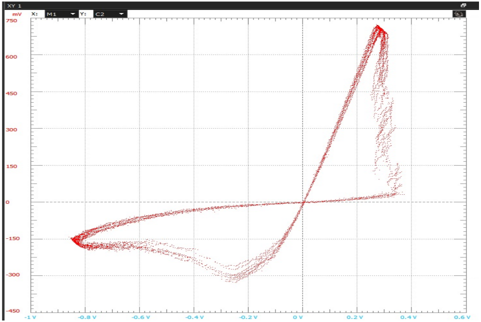
a) 

b) 

The presented graphs has been achieved using the typical HP memristor parameters:
$\mathcal{R}_{\mathrm{ON}} = 100\ \Omega$, $\mathcal{R}_{\mathrm{OFF}}/\mathcal{R}_{\mathrm{ON}} = 160$, $\mu_v = 10^{-10}\ \mathrm{cm}^2\mathrm{s}^{-1}\mathrm{V}^{-1}$, $D = 10\ \mathrm{nm}$,
$U_{max} = 1\ \mathrm{V}$.

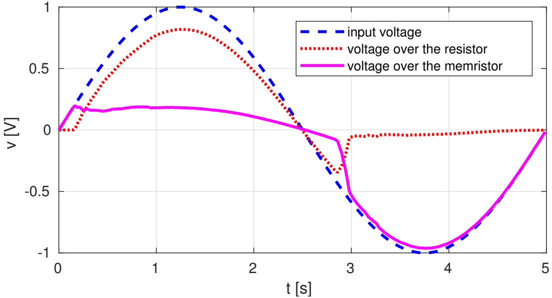
Measurments

- The memristor under study is a self-directed-channel memristor with a tungsten (W) dopant in a 16-pin ceramic DIP package fabricated by the Knowm Inc.
- The element was connected in series with the resistor $R_s = 6.2\mathrm{k}\Omega$.
- The series connection is exited by a sinusoidal voltage $v(t) = V_{\max} \sin(2\pi f t)$ wave with a given amplitude $V_{\max}$ and frequency f.



Scope print of i-v curves as an example of measured data.
($V_{\max} = 1$ V and $f = 0.5$ Hz)

The *Savitzky-Golay* filter is used to filter out high frequencies from the data. From the filtered data 400 points filling uniformly a single period of the input signal are selected as a reference data for further processing.



The time response for the input voltage $v(t) = \mathrm{V}_{\max} \sin(2\pi f t)$ (average over 6 periods), ($\mathrm{V}_{\max} = 1$ V, $f = 0.2$ Hz).

Memristor Models

Linear Ion Drift (Strukov) Model

The model is based on the assumption that oxygen ions drift with the velocity that depends linearly on the electric field (voltage) in the memristor structure. In this model, the voltage-current relation is given by

$$v(t) = (R_{\text{on}} x(t) + R_{\text{off}} (1 - x(t)))\, i(t), \qquad (1)$$

where the internal variable $x(t)$ denotes the relative width of the low-resistance region.
The dynamics of the element is defined by the following formula:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = k R_{\text{on}} i(t), \quad x \in [0, 1]. \qquad (2)$$

To make sure that $x \in [0, 1]$ one can multiply the right hand side of (2) by the *ideal rectangular window function* defined as $f(x) = 1$ for $x \in [0, 1]$ and $f(x) = 0$ outside $[0, 1]$.

Biolek Window Function

The linear model does not describe all the physical phenomena in the memristor structure.

One of the modifications is to use a different window function $f(x, i)$ to slow down the dynamics when x is close to the border of the interval $[0, 1]$. In the presentation we use the very popular window proposed by prof. Biolek.

$$f(x, i) = 1 - (x - \mathbb{1}(-i))^p \,, \tag{3}$$

where p is an even integer, and the unit step function $\mathbb{1}(\cdot)$ is defined as $\mathbb{1}(x) = 1$ for $x \geqslant 0$ and $\mathbb{1}(x) = 0$ for $x < 0$. To permit odd values of p author extend the Biolek window definition as:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = kR_{\mathrm{on}} i(t) \left(1 - |x - \mathbb{1}(-i)|^p\right) . \tag{4}$$

AGH

Asymmetric Strukov Model

Real memristors do not exhibit the symmetry in the sense that different physical phenomena define the behavior of the element depending on the polarity of the applied voltage or direction of the current. To reflect the lack of symmetry one may modify the model (2) by using different values of the parameter k for different polarities of the voltage (current direction). The asymmetric Strukov model has the following form:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \begin{cases} k_{\mathrm{on}}\ R_{\mathrm{on}}\ i(t)\ f_{\mathrm{on}}(x), & \text{for } i \geqslant 0, \\ k_{\mathrm{off}}\ R_{\mathrm{on}}\ i(t)\ f_{\mathrm{off}}(x), & \text{for } i < 0. \end{cases} \tag{5}$$

Memristor Models

VTEAM Model

Voltage controlled ThrEshold Adaptive Model is based on the voltage threshold idea. The internal variable w is confined to the interval $[w_{\mathrm{on}}, w_{\mathrm{off}}]$. In this presentation, we assume that the relation between the variable w and the resistance $R(w)$ is linear. The VTEAM model is described as

$$v(t) = \left(R_{\mathrm{on}} + \frac{w - w_{\mathrm{on}}}{w_{\mathrm{off}} - w_{\mathrm{on}}} (R_{\mathrm{off}} - R_{\mathrm{on}}) \right) i(t), \tag{6}$$

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \begin{cases} k_{\mathrm{off}} \left(\frac{v(t)}{v_{\mathrm{off}}} - 1 \right)^{\alpha_{\mathrm{off}}}, & 0 < v_{\mathrm{off}} \leqslant v, \\ 0, & v_{\mathrm{on}} < v < v_{\mathrm{off}}, \\ k_{\mathrm{on}} \left(\frac{v(t)}{v_{\mathrm{on}}} - 1 \right)^{\alpha_{\mathrm{on}}}, & v \leqslant v_{\mathrm{on}} < 0. \end{cases} \tag{7}$$

The model described by (6) and (7) is more sophisticated than the previous models. VTEAM model involves 10 parameters. Eight of them are real valued $R_{\mathrm{on}}, R_{\mathrm{off}}, w_{\mathrm{on}}, w_{\mathrm{off}}, v_{\mathrm{on}}, v_{\mathrm{off}}, k_{\mathrm{on}}, k_{\mathrm{off}} \in \mathbb{R}$ and two of them are integer valued $\alpha_{\mathrm{on}}, \alpha_{\mathrm{off}} \in \mathbb{N}$.

Fitting Data to the Models

Optimization algorithm

Let **x** be the vector of real-valued parameters of a given model and **y** be the vectors of integer-valued parameters. In the optimization problem considered the objective function is defined as

$$F(\mathbf{x},\mathbf{y}) = \sum_{j=1}^{n} \left(i_j(\mathbf{x},\mathbf{y}) - i_{\mathrm{ref},j}\right)^2, \qquad (8)$$

where n is the number of samples in the reference data, $i_{\mathrm{ref},j}$ for $j \in \{1,2,\ldots,n\}$ are the values of the current from the reference data, and $i_j(\mathbf{x},\mathbf{y})$ are the values of the current computed for a given model with the parameter values **x** and **y**.

Results

The Strukov Model with the Ideal Rectangular Window

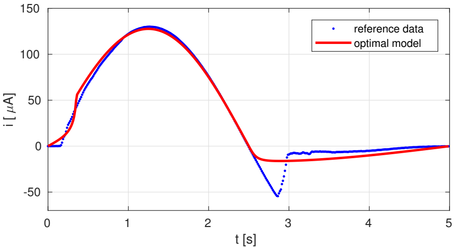
Constrains applied:

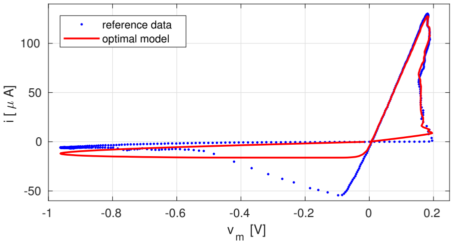
$R_{\mathrm{on}} \in [0.01, 10]$ kΩ, $R_{\mathrm{off}} \in [1, 5000]$ kΩ, $k \in [0.1, 100]$, and $x_0 \in [0, 1]$.

Initial point: $\mathbf{x}_0 = [R_{\mathrm{on}}, R_{\mathrm{off}}, k, x_0]^{\mathrm{T}} = [100, 16000, 100, 0.0]^{\mathrm{T}}$.

Min value achieved: $F = 1.0393941$

$\mathbf{x}_{\mathrm{optim}} = [R_{\mathrm{on}}, R_{\mathrm{off}}, k, x_0]^{\mathrm{T}} = [1535, 111400, 25.16, 0.8348]^{\mathrm{T}}$.





Asymmetric Strukov Model

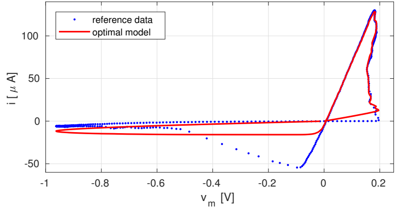
The minimized objective function value is $F = 1.02831232$

$\mathbf{x}_{\mathrm{optim}} = [R_{\mathrm{on}}, R_{\mathrm{off}}, k_{\mathrm{on}}, k_{\mathrm{off}}, x_0]^{\mathrm{T}} = [1520, 112600, 7.198, 26.07, 0.9116]^{\mathrm{T}}$.

Results

The Strukov Model with the Biolek Window

Vector $\mathbf{y} = [p]$. If $p = [1, 2, \ldots, 5] \longrightarrow F_{\mathrm{p}} = [1.153, 1.064, 1.029, 1.013, 1.006]$.
If $p \in [10, 20] \longrightarrow F \in [0.99985, 0.99997]$.
Min objective function value is $F = 0.999846521$ for $\mathbf{y}_{\mathrm{optim}} = [p] = [10]$.
$\mathbf{x}_{\mathrm{optim}} = [R_{\mathrm{on}}, R_{\mathrm{off}}, k, x_0]^{\mathrm{T}} = [1490, 1944000, 15.41, 0.9480]^{\mathrm{T}}$,



The Asymmetric Strukov Model with the Biolek Window

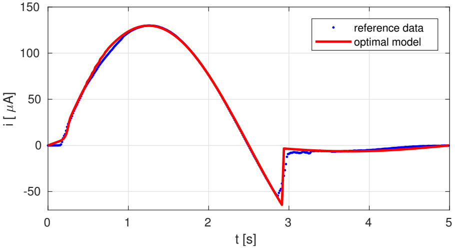
All pairs of parameters $p_{\mathrm{on}}, p_{\mathrm{off}} \in \{1, 2, \ldots, 10\}$ have been tested, then $F \in [0.996, 1.003]$. The best results are obtained for
$\mathbf{x}_{\mathrm{optim}} = [R_{\mathrm{on}}, R_{\mathrm{off}}, k_{\mathrm{on}}, k_{\mathrm{off}}, x_0]^{\mathrm{T}} = [1495, 488700, 81.23, 0.6024, 0.9975]^{\mathrm{T}}$
$\mathbf{y}_{\mathrm{optim}} = [p_{\mathrm{on}}, p_{\mathrm{off}}]^{\mathrm{T}} = [2, 7]^{\mathrm{T}}$

The VTEAM Model

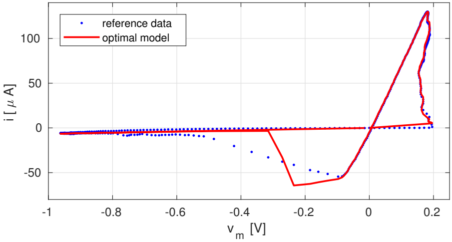
The smallest value of the objective function $F = 0.172212343$

$\mathbf{y}_{\text{optim}} = [\alpha_{\text{on}}, \alpha_{\text{off}}]^{\text{T}} = [7, 5]^{\text{T}}$

$\mathbf{x}_{\text{optim}} = [R_{\text{on}}, R_{\text{off}}, k_{\text{on}}, k_{\text{off}}, w_{\text{on}}, w_{\text{off}}, v_{\text{on}}, v_{\text{off}}, w_0]^{\text{T}} =$
$[1403, 150000, -0.005739, 11.27, 0, 0.00112, -0.1468, 0.0902, 0.000208]^{\text{T}}.$



(a) memristor current versus time



(b) the v-i plot.

Conclusions

1. Several existing memristor models have been optimized to emulate the behavior of self-directed-channel memristors with a tungsten dopant.
2. It has been shown that the VTEAM model outperforms the other models considered.
3. The standard Strukov model with the ideal rectangular window is much less successful in modeling of this device.
4. Using the Biolek window does not significantly improve the results.
5. It has been also considered an asymmetric version of the Strukov model. This modification provides only a slightly better fitting to the measurement data than the symmetric version.

AGH

www.agh.edu.pl

Thank You
For Your Attention