Modeling of Memristors Under Sinusoidal Excitations with Various Frequencies

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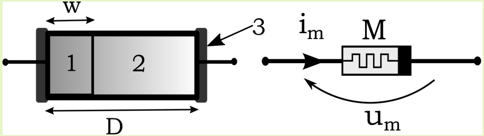
Problem description

Abstract

- In the presentation, the problem of memristors modeling is investigated.
- The elements under study are SDC (self-directed-channel) memristors with a tungsten dopant fabricated by the Knowm Inc.
- Three memristor models are considered: the asymmetric Strukov model, the Strukov model with the Biolek window, and the VTEAM model.
- Parameters of the models are fitted to experimental data using the interior-point optimization algorithm.
- The possibility of modeling memristor's behavior in a wide frequency range using the models considered is going to be presented.

Problem description

Memristor is a two-port passive element postulated by Leon Chua in 1971. The discovery of the physical solid-state structures having memristive characteristics by the HP scientists in 2008 has attracted enormous interest in the scientific community.

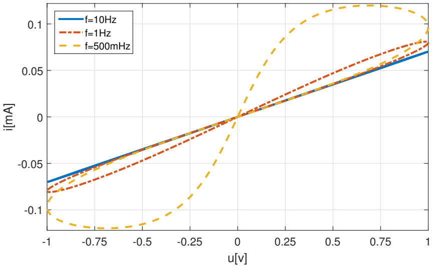


Model of the memristor and its equivalent symbol.
(1-doped region, 2-undoped region, 3-metal contact)

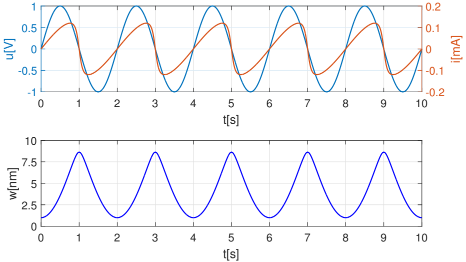
Memristor sinusoidal response

a) - The typical zero-crossing pinched hysteresis loop as the memristor response for the sinusoidal excitation ($U_{\max} = 1\mathrm{V}$).
b) - The voltage and the current in the memristor and the $w(t)$ function vs time t for $f = 0.5\mathrm{Hz}$.

a)



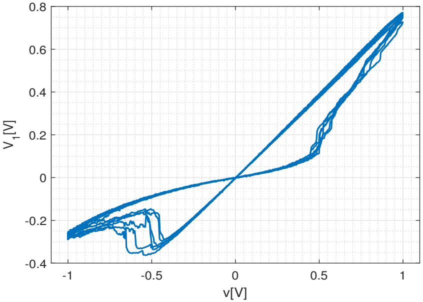
b)



The presented graphs has been achieved using the typical HP memristor parameters:
$\mathcal{R}_{\mathrm{ON}} = 100\ \Omega$, $\mathcal{R}_{\mathrm{OFF}}/\mathcal{R}_{\mathrm{ON}} = 160$, $\mu_v = 10^{-10}\ \mathrm{cm}^2\mathrm{s}^{-1}\mathrm{V}^{-1}$, $D = 10\ \mathrm{nm}$, $U_{max} = 1\ \mathrm{V}$.

Memristor Measurements

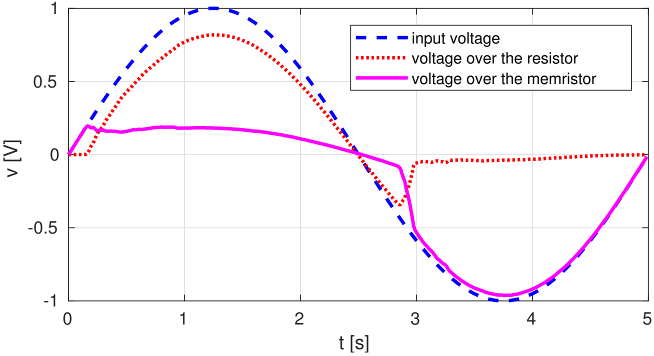
- The memristor under study is a self-directed-channel memristor with a tungsten (W) dopant in a 16-pin ceramic DIP package fabricated by the Knowm Inc.
- The element was connected in series with the resistor $R_s = 5\text{k}\Omega$.
- The series connection is exited by a sinusoidal voltage $v(t) = V_{\text{max}} \sin(2\pi f t)$ wave with a given amplitude V_{max} and frequency f.



Scope print of i-v curves as an example of measured data.
$(V_{\text{max}} = 1 \text{ V}$ and $f = 5 \text{ Hz})$

Reference Measurement Data

The *Savitzky-Golay* filter is used to filter out high frequencies from the data. From the filtered data 500 points filling uniformly a single period of the input signal are selected as a reference data for further processing.



The time response for the input voltage $v(t) = \mathrm{V}_{\max}\sin(2\pi f t)$ (average over 6 periods), $\mathrm{V}_{\max} = 1\ \mathrm{V},\ f = 0.2\ \mathrm{Hz}$.

The measurements are carried out for frequencies:
$f = 0.2\ \mathrm{Hz},\ f = 0.5\ \mathrm{Hz},\ f = 1\ \mathrm{Hz},\ f = 5\ \mathrm{Hz},\ f = 20\ \mathrm{Hz}$, and $f = 100\ \mathrm{Hz}$.

Memristor Models

Asymmetric Strukov model

The model is based on the assumption that oxygen ions drift with the velocity that depends linearly on the electric field (voltage) in the memristor structure. As real memristors do not exhibit the symmetry in the sense of the polarity of the applied voltage, some modification has been proposed.

$$v(t) = (\mathcal{R}_{\mathrm{ON}}\, x(t) + \mathcal{R}_{\mathrm{OFF}}\, (1 - x(t)))\, i(t), \tag{1}$$

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \begin{cases} k_{\mathrm{ON}}\ \mathcal{R}_{\mathrm{ON}}\ i(t)\ f(x), & \text{for } u \geqslant 0, \\ k_{\mathrm{OFF}}\ \mathcal{R}_{\mathrm{ON}}\ i(t)\ f(x), & \text{for } u < 0, \end{cases} \tag{2}$$

where $f(x)$ denotes the window function to confine the internal variable x to the interval $[0,1]$. In the original linear ion drift model the rectangular window function with $f(x) = 1$ for $x \in [0,1]$ and $f(x) = 0$ for $x \neq [0,1]$ is used. In the above equation, we use two different parameters k_{ON} and k_{OFF} to define the rate of changing the internal variable depending of the sign of u. The model presented in this section depends of four parameters: k_{ON}, k_{OFF}, $\mathcal{R}_{\mathrm{ON}}$, and $\mathcal{R}_{\mathrm{OFF}}$.

Memristor Models

Biolek Window Function

The linear model does not describe all the physical phenomena in the memristor structure.
One of the modifications is to use a different window function $f(x,i)$ to slow down the dynamics when x is close to the border of the interval $[0,1]$. In the presentation we use the very popular window proposed by prof. Biolek.

$$f(x,i) = \begin{cases} 1 - |x - \mathrm{step}(-i)|^p, & \text{for } x \in [0,1], \\ 0, & \text{for } x \notin [0,1], \end{cases} \tag{3}$$

where p is an integer. The main role of the window function (3) is to force the variable x to stay in the interval $[0,1]$ and to slow down the dynamics when x is close to the border of this interval.
Using this window function introduces an additional integer parameter p.
The asymmetric Strukov model with the asymmetric Biolek window depends on six parameters: k_{ON}, k_{OFF}, $\mathcal{R}_{\mathrm{ON}}$, $\mathcal{R}_{\mathrm{OFF}}$, p_{ON} and p_{OFF}.

Memristor Models

VTEAM model

In the VTEAM model, the internal state variable w is confined to the interval $[w_{\mathrm{ON}}, w_{\mathrm{OFF}}]$. The VTEAM model is defined by:

$$v(t) = \frac{\mathcal{R}_{\mathrm{ON}}\left(w - w_{\mathrm{OFF}}\right) + \mathcal{R}_{\mathrm{OFF}}\left(w_{\mathrm{ON}} - w\right)}{w_{\mathrm{ON}} - w_{\mathrm{OFF}}} i(t) \tag{4}$$

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \begin{cases} k_{\mathrm{ON}}\left(\frac{u(t)}{v_{\mathrm{ON}}} - 1\right)^{\alpha_{\mathrm{ON}}} & 0 < v_{\mathrm{ON}} < v \\ 0, & v_{\mathrm{OFF}} < v < v_{\mathrm{ON}} \\ k_{\mathrm{OFF}}\left(\frac{v(t)}{v_{\mathrm{OFF}}} - 1\right)^{\alpha_{\mathrm{OFF}}} & u < v_{\mathrm{OFF}} < 0 \end{cases} \tag{5}$$

For $v \in [v_{\mathrm{OFF}}, v_{\mathrm{ON}}]$ the state variable remains constant (voltage threshold idea).
The VTEAM model depends on ten parameters. Eight of them are real valued ($\mathcal{R}_{\mathrm{ON}}$, $\mathcal{R}_{\mathrm{OFF}}$, v_{ON}, v_{OFF}, k_{ON}, k_{OFF}, w_{ON}, and w_{OFF}) while two of them are integer valued (α_{ON} and α_{OFF}).

Memristor Models

VTEAM model (continued)

$$v(t) = \frac{\mathcal{R}_{\mathrm{ON}}\left(w - w_{\mathrm{OFF}}\right) + \mathcal{R}_{\mathrm{OFF}}\left(w_{\mathrm{ON}} - w\right)}{w_{\mathrm{ON}} - w_{\mathrm{OFF}}} i(t) \tag{6}$$

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t} = \begin{cases} k_{\mathrm{ON}}\left(\frac{u(t)}{v_{\mathrm{ON}}} - 1\right)^{\alpha_{\mathrm{ON}}} & 0 < v_{\mathrm{ON}} < v \\ 0, & v_{\mathrm{OFF}} < v < v_{\mathrm{ON}} \\ k_{\mathrm{OFF}}\left(\frac{v(t)}{v_{\mathrm{OFF}}} - 1\right)^{\alpha_{\mathrm{OFF}}} & u < v_{\mathrm{OFF}} < 0 \end{cases} \tag{7}$$

Note that by shifting the internal variable $w' = w - w_{\mathrm{ON}}$, $w'_{\mathrm{OFF}} = w_{\mathrm{OFF}} - w_{\mathrm{ON}}$ one can force the parameter w_{ON} to be zero. For the case of the ideal rectangular window by rescaling w one can achieve any value of w_{OFF}. It follows that one can fix the values of parameters w_{ON} and w_{OFF} without losing the generality of the model. In the optimization process we set $w_{\mathrm{ON}} = 0$ and $w_{\mathrm{OFF}} = 10^{-3}$.

Fitting Data to the Models

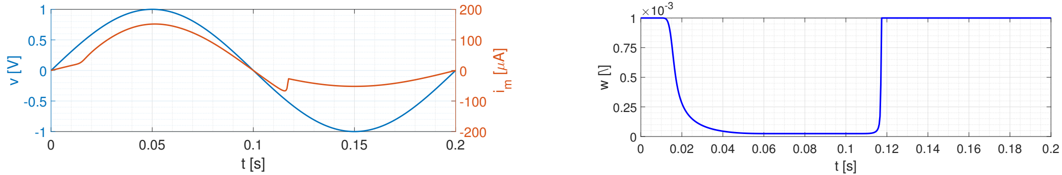
Optimization procedure

Let **x** be the vector of real-valued parameters of a given model and **y** be the vectors of integer-valued parameters. The cost function is defined as:

$$F(\mathbf{x},\mathbf{y}) = \frac{\sum_{j=1}^{n} \left(i_j(\mathbf{x},\mathbf{y}) - i_{\mathrm{ref},j}\right)^2}{\sum_{j=1}^{n} i_{\mathrm{ref},j}^2}, \tag{8}$$

where n is the number of samples in the reference data, $i_{\mathrm{ref},j}$ for $j \in \{1, 2, \dots n\}$ are the values of the current from the reference data, and $i_j(\mathbf{x},\mathbf{y})$ are the values of the current computed for a given model with parameter values $\mathbf{x},\mathbf{y}$. During the computations, the reference voltage $v_{\mathrm{ref},j}$ for $j \in \{1, 2, \dots n\}$ is applied as the input signal.
The optimization goal is to find parameter values $\mathbf{x},\mathbf{y}$ minimizing the cost function $F(\mathbf{x},\mathbf{y})$.
Real-valued and integer-valued parameters are handled in a different way. For fixed values of integer-valued parameters **y** finding the optimum values of the real-valued parameters **x** is carried out using the *interior-point* algorithm. This process is repeated for all admissible combinations of integer-valued parameters.

Results (example VTEAM)

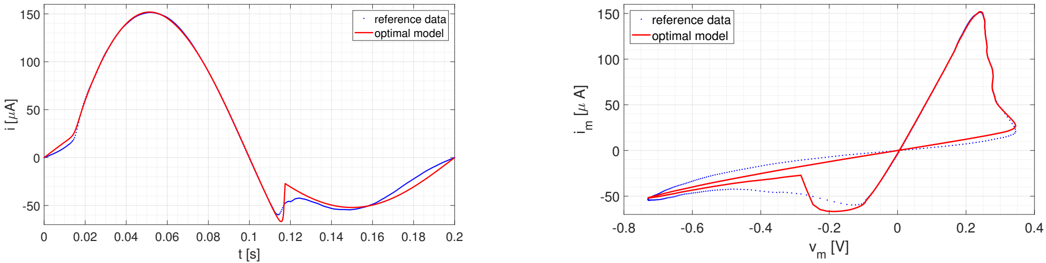


Time responses of the optimized VTEAM model to sinusoidal excitation with the frequency $f = 5\ \mathrm{Hz}$.

Time waveforms are presented for the best obtained parameter values of the optimization problem:
$\mathcal{R}_{\mathrm{ON}} = 1.22\ \mathrm{k\Omega}$, $\mathcal{R}_{\mathrm{OFF}} = 16.529\ \mathrm{k\Omega}$, $k_{\mathrm{ON}} = -2.34\ \mathrm{m/s}$, $k_{\mathrm{OFF}} = 204.02\ \mathrm{nm/s}$,
$v_{\mathrm{ON}} = -200.5\ \mathrm{mV}$, $v_{\mathrm{OFF}} = 15.4\ \mathrm{mV}$, $\alpha_{\mathrm{ON}} = 5$, $\alpha_{\mathrm{OFF}} = 6$
Value of the cost function is $F(\mathbf{x}, \mathbf{y}) = 3.137 \cdot 10^{-3}$.
Internal variable is periodic, i.e. $x(0) = x(T)$, where $T = 0.2\ \mathrm{s}$ is the period of the input signal.

Results (example VTEAM)

Comparision beetween VTEAM model and the reference data



Comparison of current response of and the v-i plots of the optimized VTEAM model with the reference data; $f = 5\ \mathrm{Hz}$.

Similar computations have been carried out for other considered memristor models. For example, for the asymmetric Strukov model with the Biolek window the best value of the cost function is approximately six times worse.

Model Performance with Varying Frequency

	frequency [Hz]					
	0.2	0.5	1	5	20	100
$\mathcal{R}_{\mathrm{ON}}$	1.1822	1.6014	1.8163	2.0517	3.5509	5.0631
$\mathcal{R}_{\mathrm{OFF}}$	15.009	17.396	12.000	3850.9	1739.7	320.34
k_{ON}	99.964	99.998	100.00	100.00	99.999	99.998
k_{OFF}	60.671	99.989	100.00	0.46583	1.3469	7.1125
p_{ON}	3	10	10	10	10	7
p_{OFF}	10	10	10	5	7	7
$F(\mathbf{x}, \mathbf{y})$	0.00408	0.01660	0.01872	0.02015	0.09044	0.22610

Optimal parameters values of the asymmetric Strukov model with Biolek window for different frequencies ($\mathcal{R}_{\mathrm{ON}}$ and $\mathcal{R}_{\mathrm{OFF}}$ are in $\mathrm{k}\Omega$)

	frequency [Hz]					
	0.2	0.5	1	5	20	100
$\mathcal{R}_{\mathrm{ON}}$	0.49884	1.1804	1.3620	1.2221	2.0852	1.1985
$\mathcal{R}_{\mathrm{OFF}}$	16.850	22.762	15.082	14.200	16.529	25.331
k_{ON}	-2.3568	-0.097703	-43.049	-2.3408	-9.5335	-17.134
k_{OFF}	401.35	151.69	1702.0	204.02	16630.0	6435.0
v_{ON}	-149.19	-136.93	-183.73	-200.48	-223.86	-251.51
v_{OFF}	16.387	7.0533	6.8630	15.390	12.659	0.31765
α_{ON}	7	7	7	5	8	10
α_{OFF}	7	5	3	6	1	1
$F(\mathbf{x}, \mathbf{y})$	0.00078	0.00125	0.00177	0.00313	0.00210	0.00174

Optimal parameters values of the asymmetric model for different frequencies with VTEAM model ($\mathcal{R}_{\mathrm{ON}}$ and $\mathcal{R}_{\mathrm{OFF}}$ are in $\mathrm{k}\Omega$, k_{ON} in $\mathrm{m/s}$, k_{OFF} in $\mathrm{nm/s}$, v_{ON} and v_{OFF} are in mV)

Model Performance with Varying Frequency

f[Hz]	0.2	0.5	1	5	20	100
0.2	0.00408	0.00848	0.01921	0.05291	0.12080	0.20077
0.5	0.03004	0.01660	0.03104	0.02423	0.08648	0.16766
1	0.06102	0.03968	0.01872	0.02583	0.09116	0.17184
5	0.15636	0.25106	0.18276	0.02015	0.06105	0.14387
20	0.20349	0.23243	0.19173	0.14922	0.09044	0.14741
100	0.33462	0.24644	0.25097	0.41828	0.25549	0.22610

Results comparison for different frequencies in case of the asymmetric Strukov model.

f[Hz]	0.2	0.5	1	5	20	100
0.2	0.00078	0.00730	0.00435	0.00629	0.02790	0.05052
0.5	0.01008	0.00125	0.01242	0.01830	0.02637	0.04050
1	0.00529	0.01575	0.00177	0.00332	0.02223	0.05113
5	0.01236	0.02288	0.00841	0.00313	0.01928	0.05205
20	0.04315	0.03395	0.08297	0.02129	0.00210	0.02047
100	0.11434	0.07271	0.43133	0.07007	0.04546	0.00174

Results comparison for different frequencies in case of the VTEAM model.

Conclusions

1. Several existing memristor models have been fitted to model behaviors of a self-directed-channel memristor with tungsten dopant.
2. It has been shown that the VTEAM model outperforms the other models considered.
3. It has been shown that neither of the models considered is suitable to model behavior of self-directed-channel memristors in a wide frequency range.
4. Future work will include testing other models of memristors.
5. We also plan to apply multi-objective optimization approach to improve the performance of various models in a wider frequency range.

Thank You
For Your Attention