Nonlinear dynamics of coupled inductor-capacitor-memristor oscillators

Zbigniew Galias, galias@agh.edu.pl

Department of Electrical Engineering, AGH University of Science and Technology, Poland

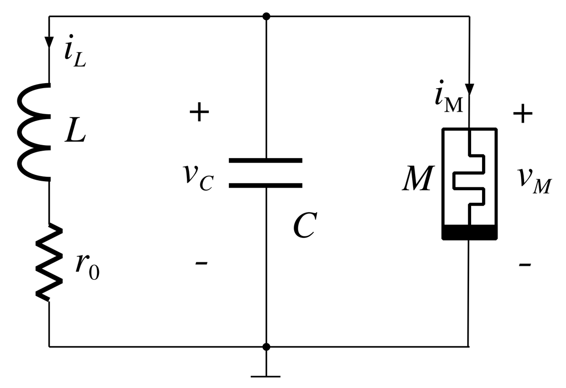
Introduction

- Memristors have received significant attention due to a number of possible applications including large capacity non-volatile memories and neuromorphic systems.
- Design of memristor based neuromorphic systems requires a thorough understanding of the nonlinear dynamics of coupled memrsistor based oscillators.

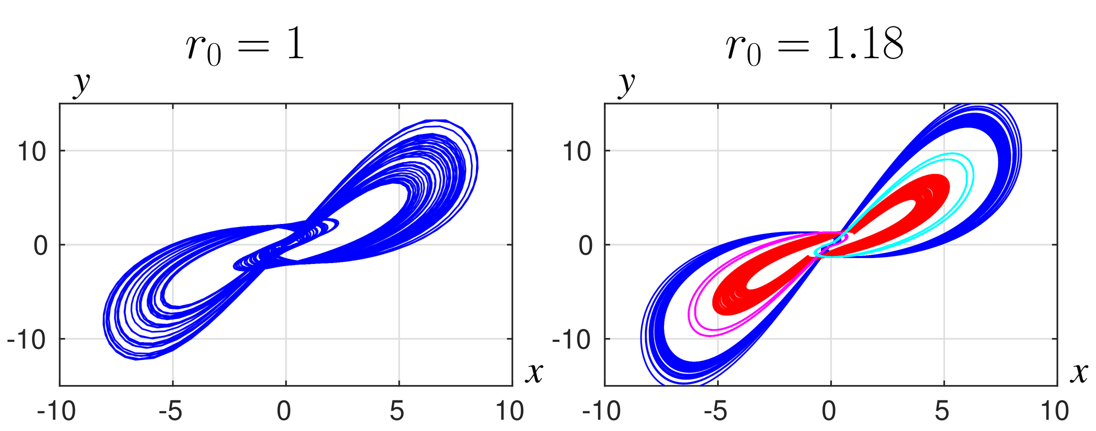
Objectives

- Study of the dynamics and the stability of synchronized motions in coupled memristor based oscillators.

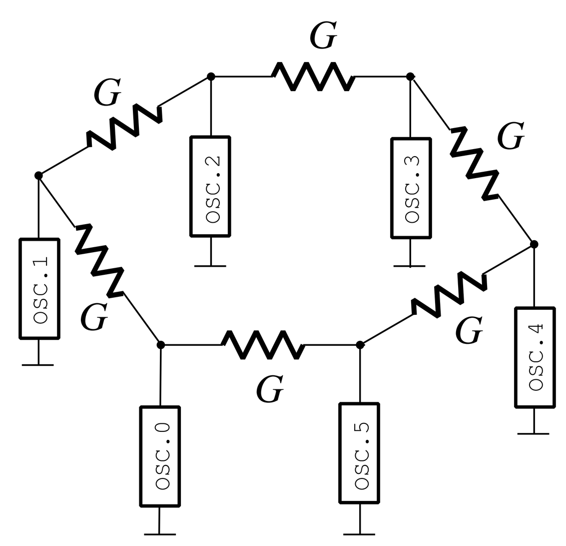
Inductor-capacitor-memristor circuit



- Circuit equations:
$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix} = F\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}(y-r_0x)/L\\ (-x-\alpha yz)/C\\ -\beta+y^2\end{pmatrix},$$
where $x=i_L$, $y=v_C$, and z is the internal variable of the memristor.
- Parameters: $L=0.025$, $C=0.025$, $\alpha=0.825$, and $\beta=10$, $r_0=1$ or $r_0=1.18$
- A single chaotic attractor for $r_0=1$, two chaotic and two periodic attractors for $r_0=1.18$:



Ring of coupled oscillators



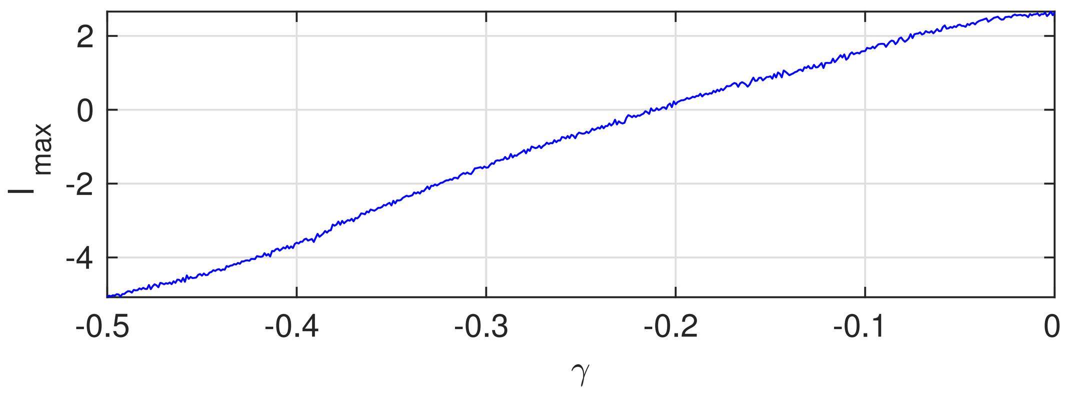
- Equations of coupled oscillators
$$\dot{w}_k = F(w_k) + GE\sum_{j=0}^{n-1} H_{kj}w_j, \quad E = \begin{pmatrix}0&0&0\\ 0&C^{-1}&0\\ 0&0&0\end{pmatrix},$$
where $w_k=(x_k,y_k,z_k)^\top$ and H is the coupling matrix.
- For $n=2$ the coupling matrix is
$$H=\begin{pmatrix}-1&1\\ 1&-1\end{pmatrix}.$$
- For $n>2$ the coupling coefficients are
$$H_{kj}=\begin{cases}-2 & \text{for } k=j,\\ 1 & \text{for } |k-j|\in\{1,n-1\},\\ 0 & \text{otherwise.}\end{cases}$$

Master Stability Function Approach

- The generalized variational equation
$$\dot{w}=F(w), \quad \dot{\xi}=(F'(w)+\gamma E)\xi.$$
- Study the generalized variational equation as a function of γ.
- Identify the region $\Gamma\subset\mathbb{R}$ such that for $\gamma\in\Gamma$ all Lyapunov exponents are negative.
- The synchronous motion is stable if $\gamma_k=G\lambda_k\in\Gamma$ for $k=1,2,\ldots,n-1$, where λ_k are nonzero eigenvalues of the coupling matrix H.

Master Stability Function Analysis

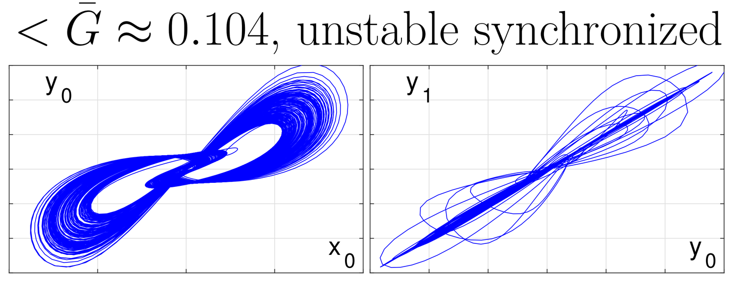
- For $r_0=1$ there exists a single chaotic attractor.
- The maximum Lyapunov exponent $l_{\max}$ of the generalized variational equation for $r_0=1$ versus γ:



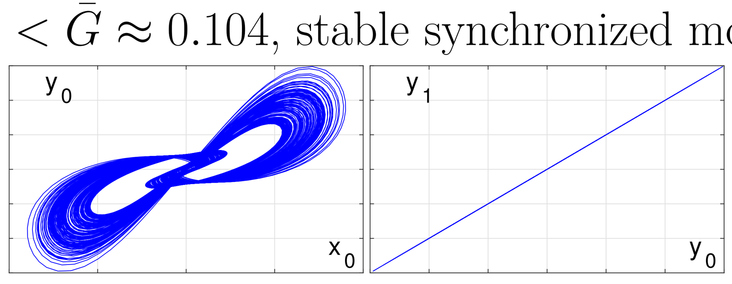
- For $\gamma<\bar{\gamma}\approx-0.208$ the maximum Lyapunov exponent is negative.
- For $n=2$ the eigenvalues of the coupling matrix H are $\lambda_0=0$, $\lambda_1=-2$, the synchronous chaotic motion is stable if $G>\bar{\gamma}/\lambda_1=\bar{G}\approx 0.104$.
- For $n=3$ the eigenvalues of H are $\lambda_0=0$ and $\lambda_1=\lambda_2=-3$, the synchronous chaotic motion is stable if $G\geq\bar{\gamma}/\lambda_1=\bar{G}\approx 0.0693$.
- Generally, for $n\geq 3$ the eigenvalues of H are $\lambda_k=-4\sin^2(\pi k/n)$, λ_1 has the smallest nonzero absolute value, the stability condition is
$$G\geq\bar{\gamma}\lambda_1^{-1}=-0.25\bar{\gamma}/\sin^2(\pi/n).$$

Simulation results, $r_0=1$, $n=2$

- $G=0.10<\bar{G}\approx 0.104$, unstable synchronized motion

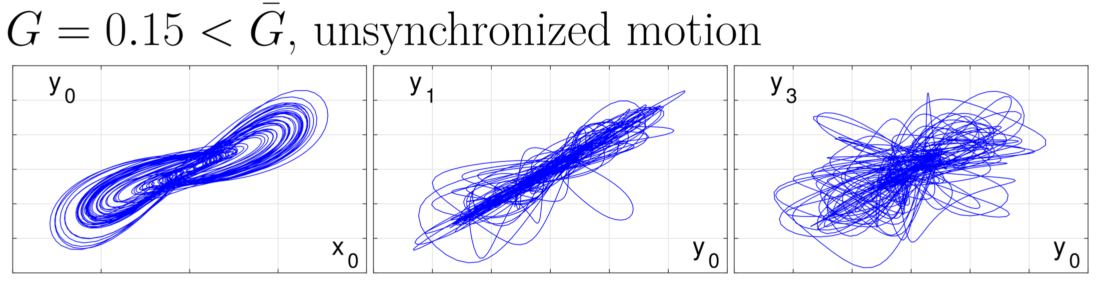


- $G=0.11<\bar{G}\approx 0.104$, stable synchronized motion

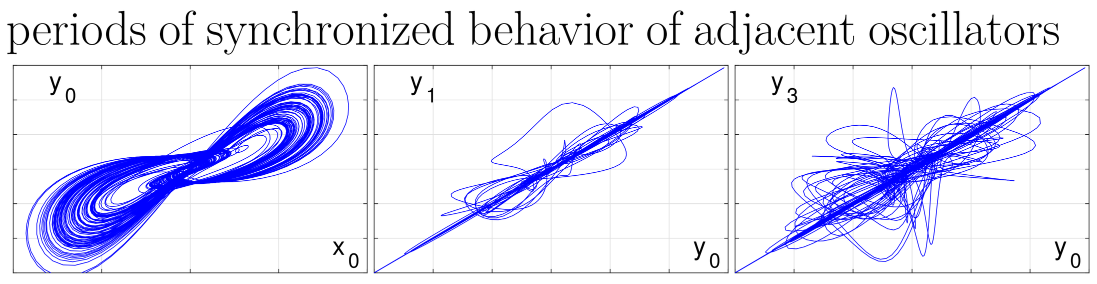


Simulation results, $r_0=1$, $n=6$

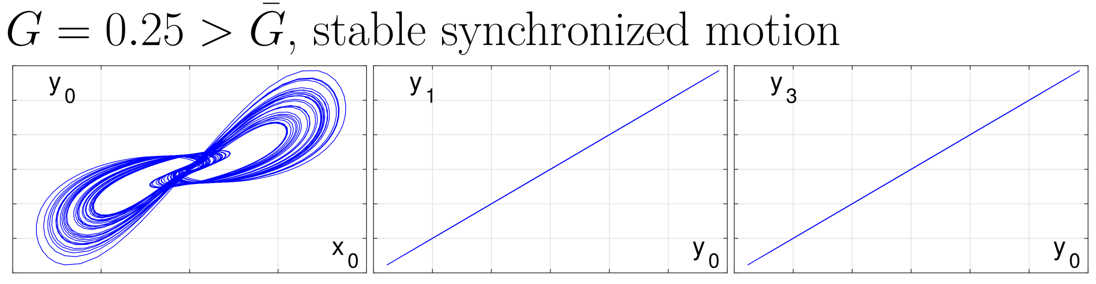
- From the stability function analysis it follows that the synchonous chaotic motion is stable for $G>\bar{G}\approx 0.208$.
- $G=0.15<\bar{G}$, unsynchronized motion



- $G=0.2<\bar{G}$, periods of synchronized behavior of adjacent oscillators

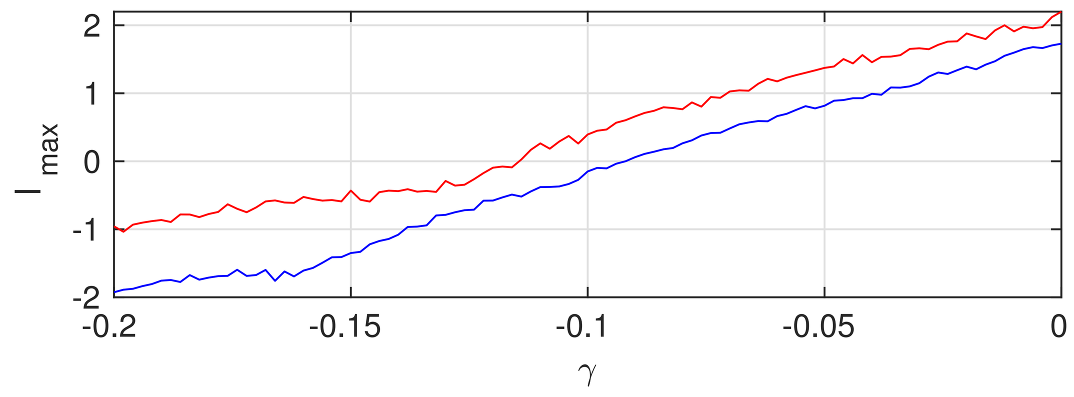


- $G=0.25>\bar{G}$, stable synchronized motion



Master Stability Function Analysis, $r_0=1.18$

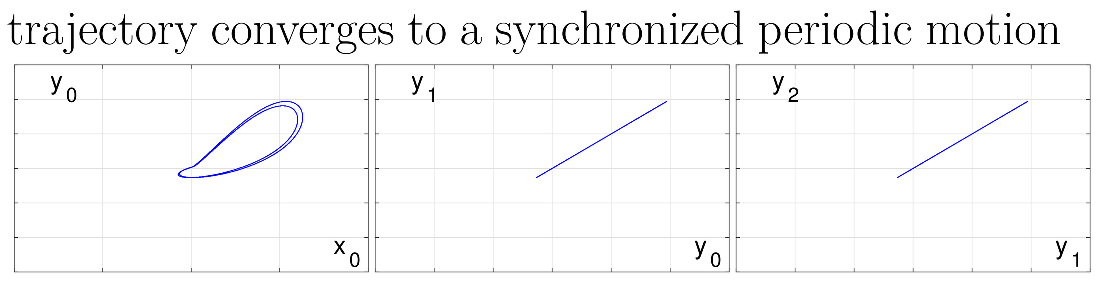
- There exists two self-symmetric chaotic attractors and a symmetric pair of periodic attractors.
- The maximum Lyapunov exponent $l_{\max}$ of the generalized variational equation for $r_0=1.18$ versus γ:



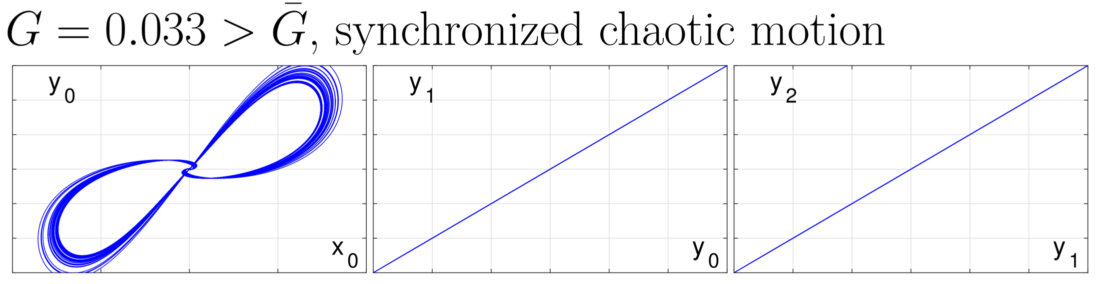
- Blue plot (the larger attractor): for $\gamma<\bar{\gamma}\approx-0.092$ the maximum Lyapunov exponent is negative.
- Red plot (the smaller attractor): for $\gamma<\bar{\gamma}\approx-0.114$ the maximum Lyapunov exponent is negative.

Simulation results, $r_0=1.18$, $n=3$

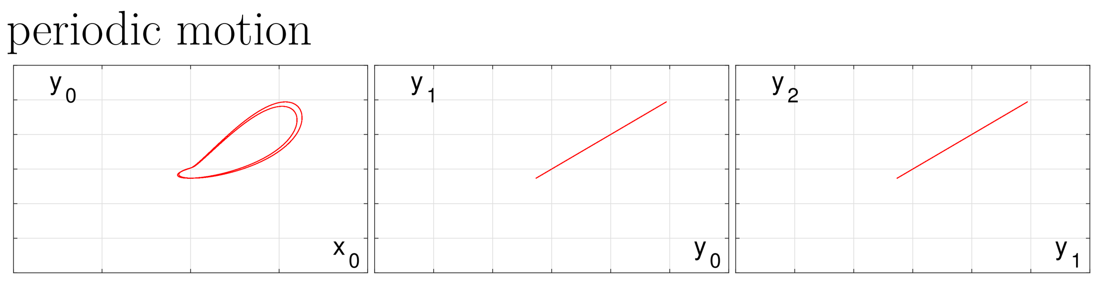
- Large amplitude chaotic motion.
- From the stability function analysis it follows that the synchonous chaotic motion is stable for $G>\bar{G}\approx 0.0307$.
- $G=0.03<\bar{G}$, unstable synchronized chaotic motion, trajectory converges to a synchronized periodic motion



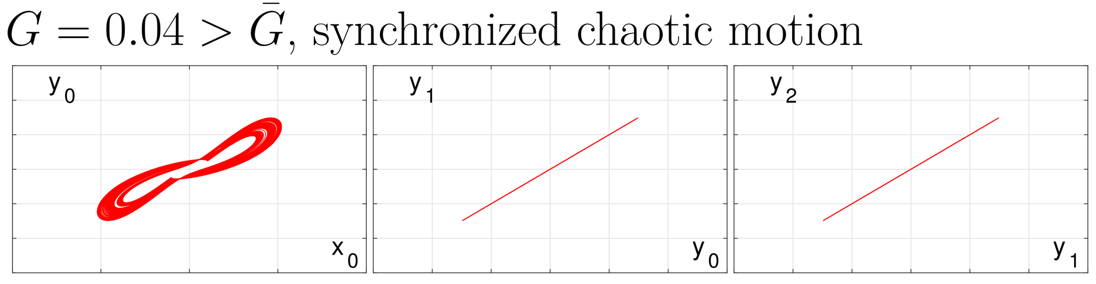
- $G=0.033>\bar{G}$, synchronized chaotic motion



- Small amplitude chaotic motion.
- From the stability function analysis it follows that the synchonous chaotic motion is stable for $G>\bar{G}\approx 0.038$.
- $G=0.033<\bar{G}$, unstable synchronized chaotic motion, trajectory converges to a synchronized periodic motion



- $G=0.04>\bar{G}$, synchronized chaotic motion



Conclusions

- Dynamics of coupled inductor-capacitor-memristor oscillators has been studied.
- Synchronous chaotic motions have been observed.
- Stability conditions have been derived using the master stability function approach.
- Results based on the master stability function approach have been confirmed in simulations.
- It has been shown that stability conditions for coexisting chaotic attractors may be different.
- For a given coupling strength one of the synchronous chaotic motions can be stable while the other may be unstable.

Acknowledgements

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