Study of Amplitude Control and Dynamical Behaviors of a Memristive Band Pass Filter Circuit

Zbigniew Galias
Department of Electrical Engineering
AGH University of Science and Technology

May 30th, 2018, ISCAS, Florence

Plan of the talk

- Third Order Memristive Band Pass Filter Circuit.
- Total and Partial Amplitude Control (TAC, PAC).
- A Three Parameter Model of the Circuit.
- Analysis of the Three Parameter Model.
- Conclusions.

Memristors

- A classical memristor is a circuit element characterized by a nonlinear relation between charge and flux. Its resistance depends on the history of current flowing through the element or the voltage across the element.
- Memristors were theoretically introduced in 1971.
- Memristive behavior was recognized in a nano-scale double-layer $\mathrm{TiO_2}$ film in 2008.
- Memristors have received significant attention due to a number of possible applications including large-capacity nonvolatile memories and neuromorphic systems.
- It has been confirmed that memristors are useful for designing nonlinear oscillators.
- "A simple third-order memristive band pass filter chaotic circuit", (B. Bao, N. Wang, Q. Xu, H. Wu, and Y. Hu, IEEE Trans. Circ. Syst. II, vol. 64, no. 8, 2017).

An extended voltage controlled memristor

- An extended voltage controlled memristor:

$$i = G(z, v)v, \quad \frac{dz}{dt} = f(z, v),$$

  where $z \in \mathbb{R}^n$ is the vector of internal state variables and $G(z, u)$ is called the memductance.

- Dynamics of the memristor used in the band pass filter circuit:

$$\begin{aligned} i &= R_c^{-1}(1 - gv_0^2)v \\ \dot{v}_0 &= -(R_b C_0)^{-1} v_0 - (R_a C_0)^{-1} v, \end{aligned}$$

  where i, v, and v_0 are the memristor's current, voltage, and internal variable, respectively.

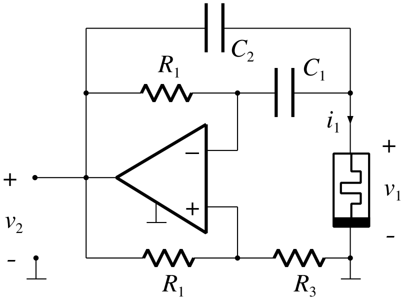
- This memristor can be implemented using two op-amps, three resistors, one capacitor, and two multipliers.

Third Order Memristive Band Pass Filter Circuit

- The dynamics of the circuit is defined by:

$$\dot{v}_0 = -(R_b C_0)^{-1} v_0 - (R_a C_0)^{-1} v_1,$$
$$\dot{v}_1 = (1 - g v_0^2)\big((k-1) R_c C_2\big)^{-1} v_1 - R_1^{-1}(C_1^{-1} + k^{-1} C_2^{-1}) v_2,$$
$$\dot{v}_2 = k(1 - g v_0^2)\big((k-1) R_c C_2\big)^{-1} v_1 - R_1^{-1}(C_1^{-1} + C_2^{-1}) v_2,$$

- v_0 is the memristor's internal variable,
- v_1 is the voltage across the memristor,
- v_2 is the voltage between the ground and the output of the operational amplifier,
- $k = 1 + R_2/R_3$.



Third Order Memristive Band Pass Filter Circuit

- Notations: $x = v_0$, $y = v_1$, $z = v_2$,
- rescaling time $\tau = t/(R_1 C_1)$,
- parameters: $\alpha = C_2/C_1$, $\delta = (R_1 C_2)/(R_b C_0)$, $\varrho = (R_1 C_2)/(R_a C_0)$, $\varepsilon = R_1/R_c$,
- the circuit equations can be rewritten as

$$\begin{aligned}
\dot{x} &= -\delta x - \varrho y, \\
\dot{y} &= \varepsilon(1 - gx^2)(k-1)^{-1} y - (\alpha + k^{-1}) z, \\
\dot{z} &= k\varepsilon(1 - gx^2)(k-1)^{-1} y - (\alpha + 1) z.
\end{aligned}$$

- behavior of the system depends on six parameters: δ, ϱ, g, k, ε and α,
- reference: "A simple third-order memristive band pass filter chaotic circuit", (B. Bao, N. Wang, Q. Xu, H. Wu, and Y. Hu, IEEE Trans. Circ. Syst. II, vol. 64, no. 8, 2017).

Amplitude Control Mechanisms

- Total Amplitude Control (TAC):
  - a single parameter is modified,
  - all variables are rescaled,
  - in TAC, it is required that the right-hand side of the equation defining the dynamical system contain terms which are all monomials of the same degree apart from one, whose coefficient can be used for amplitude control.
- Partial Amplitude Control (PAC):
  - a single parameter is modified,
  - amplitudes of selected variables are rescaled, while amplitudes of other variables are not altered.
- Amplitude control (partial and total) permits adjusting amplitudes of generated signals to a desired level without the necessity of using amplifiers.

Total amplitude control

- Six linear terms and two nonlinear terms, the nonlinear terms are of the same degree and both contain the parameter g, which is not present in other terms,
- it follows that the total amplitude control is possible,
- variable change: $(x, y, z) \mapsto (sx, sy, sz)$,
- system's equations in the new variables:

$$\begin{aligned}
\dot{x} &= -\delta x - \varrho y, \\
\dot{y} &= \varepsilon(1 - s^{-2} g x^2)(k-1)^{-1} y - (\alpha + k^{-1}) z, \\
\dot{z} &= k\varepsilon(1 - s^{-2} g x^2)(k-1)^{-1} y - (\alpha + 1) z,
\end{aligned}$$

- the only difference: g is replaced by g/s^2,
- conclusion: changing g to g/s scales trajectories by the factor $\sqrt{s}$ in all variables,
- a total amplitude control.

Partial amplitude control

- Variable change: $(x, y, z) \mapsto (x, sy, sz)$,
- system's equations in the new variables:

$$\begin{aligned} \dot{x} &= -\delta x - s^{-1} \varrho y, \\ \dot{y} &= \varepsilon(1 - gx^2)(k-1)^{-1} y - (\alpha + k^{-1}) z, \\ \dot{z} &= k\varepsilon(1 - gx^2)(k-1)^{-1} y - (\alpha + 1) z, \end{aligned}$$

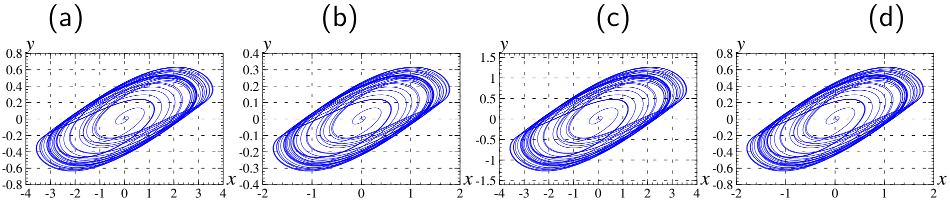
- the only difference: ϱ is replaced by ϱ/s,
- conclusion: changing ϱ to ϱ/s is equivalent to rescaling of variables y and z while the variable x is not changed,
- a partial amplitude control regarding variables y and z.

Partial amplitude control with two parameters

- The combination of two parameter changes used before:
  - ϱ is changed to ϱ/s,
  - g is changed to g/s^2,
- results:
  - rescaling x by the factor s,
  - the amplitudes of y and z are unaltered,
- this is a partial amplitude control regarding the variable x,
- independent changing of ϱ and g: we may independently control the amplitude of x and the amplitudes of y and z.

Partial and Total Amplitude Control: Examples

- $\delta = 8$, $\varepsilon = 500/3$, $k = 21$, $\alpha = 1$
- (a) $\varrho = 80$, $g = 0.1$, $(x_0, y_0, z_0) = (0.1, 0, 0.1)$,
- (b) $\varrho = 80$, $g = 0.4$, $(x_0, y_0, z_0) = (0.05, 0, 0.05)$, TAC,
- (c) $\varrho = 40$, $g = 0.1$, $(x_0, y_0, z_0) = (0.1, 0, 0.2)$, PAC $(y,\ z)$,
- (d) $\varrho = 40$, $g = 0.4$, $(x_0, y_0, z_0) = (0.05, 0, 0.1)$, PAC (x).



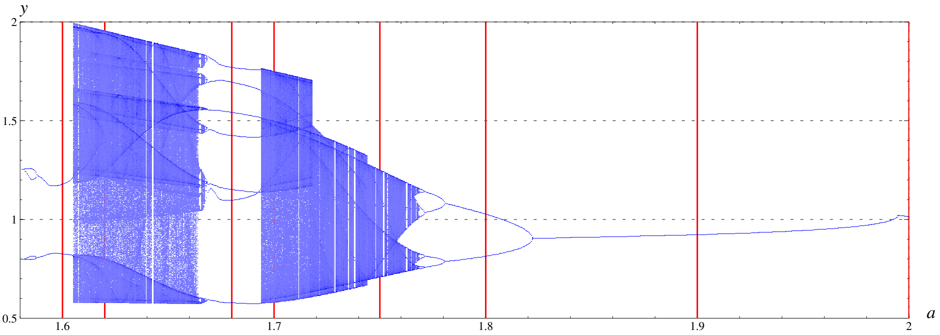
A Three Parameter Model of the Circuit

- Notations: $a = R_b C_0(k\alpha + 1)/(R_1 C_2) - R_b C_0/((k-1)R_c C_2)$, $b = R_b C_0(k\alpha + 1)/(R_1 C_2)$, $c = R_b C_0(k-1)/(R_1 C_1)$, $s = \sqrt{(k-1)R_c C_2/(gR_b C_0)}$,
- new variables $x = -v_0/s$, $y = v_1 R_b/(sR_a)$, $w = v_2 R_b/(skR_a)$, $z = y - w$, $\tau = t/(R_b C_0)$,
- dynamical system in variables x, y, and z:

$$\begin{aligned}\dot{x} &= -x + y,\\ \dot{y} &= -ay + bz - x^2 y,\\ \dot{z} &= c(z - y),\end{aligned}$$

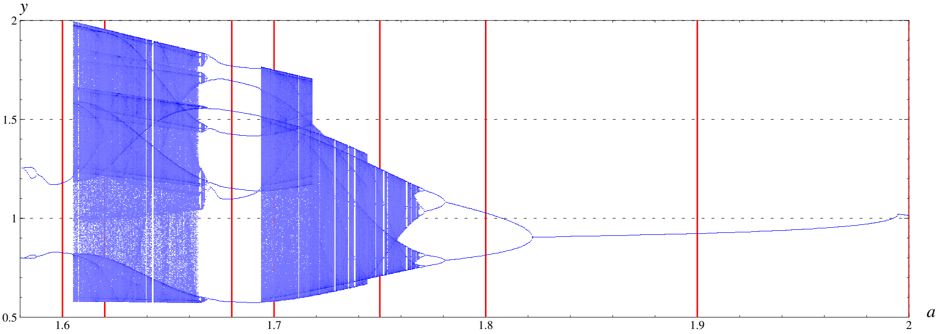
- we obtain an equivalent dynamical system with three parameters,
- reducing the number of parameters simplifies the process of studying dynamics of the system.

- The system is symmetric with respect to the transformation $(x, y, z) \mapsto (-x, -y, -z)$,
- the system possesses three equilibria $(0, 0, 0)$, and $\pm x^\star = (\pm x_1^\star, \pm x_1^\star, \pm x_1^\star)$, where $x_1^\star = \sqrt{b - a}$,
- bifurcation diagram for $a \in [1.58, 2]$, $b = 2.75$, $c = 2.5$:



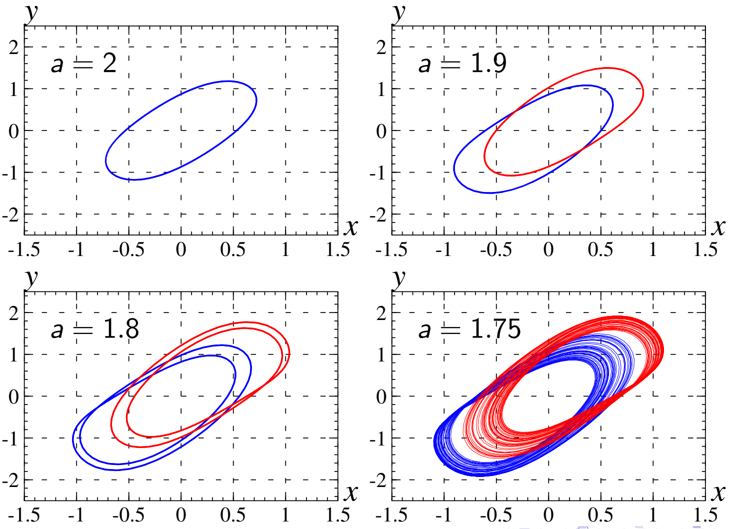
Analysis of the Three Parameter Model

- the system is symmetric with respect to the transformation $(x, y, z) \mapsto (-x, -y, -z)$,
- the system possesses three equilibria $(0, 0, 0)$, and $\pm x^\star = (\pm x_1^\star, \pm x_1^\star, \pm x_1^\star)$, where $x_1^\star = \sqrt{b - a}$,
- bifurcation diagram for $a \in [1.58, 2]$, $b = 2.75$, $c = 2.5$



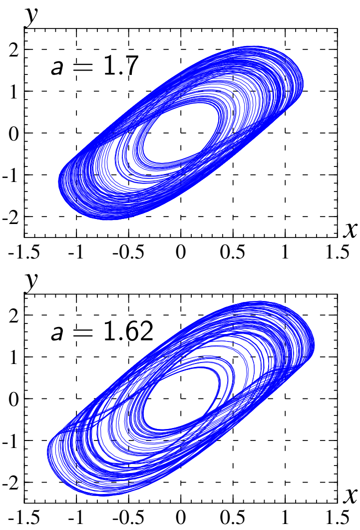
Attractors existing for selected values of a

- $b = 2.75$, $c = 2.5$, $(x_0, y_0, z_0) = (\pm 0.1, 0, \pm 0.1)$.



Attractors existing for selected values of a

- $b = 2.75$, $c = 2.5$, $(x_0, y_0, z_0) = (\pm 0.1, 0, \pm 0.1)$.



Rigorous confirmation of simulation results

- The existence of attractors is confirmed using interval arithmetic tools.
- Computations in interval arithmetic are carried out in such a way that a proper rounding is used when calculating bounds of mathematical expressions.
- Computation are carried out for the return map defined by the section $\{(x, y, z) \colon x = 0\}$.
- The return map P and its derivative are rigorously evaluated using the CAPD library.
- The existence of periodic attractors is verified using the interval Newton method.
- For chaotic attractors the existence of infinitely many periodic orbits and chaotic trajectories is confirmed using the method of covering relations.

Conclusions

- It has been shown that a memristive band pass filter circuit possesses a total and partial amplitude control mechanisms.
- In the total amplitude control by changing the value of one of the circuit's parameters the amplitudes of all variables are rescaled by the same factor.
- In the partial amplitude control the amplitude of one of the variables remains constant while the others are rescaled by the same factor.
- A linear change of coordinates has been constructed in such a way that the behavior of the resulting dynamical system depends on three parameters only.
- Dynamical phenomena in this system have been studied.