On Multiobjective Optimization of Switch Allocation in Radial Distribution Networks

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Plan of the talk

- Research goal.
- Radial distribution networks: assumptions and notations.
- Performance indexes: SAIFI, SAIDI, and AENS.
- Multiobjective optimization problem.
- Optimization algorithms:
  - exhaustive search (ES) method,
  - evolutionary algorithm (EA),
  - tree structure based (TS) algorithm.
- Results
  - Test distribution network.
  - Computational complexity.
  - Optimization results.
- Conclusions.

Research goal

- Reducing the consequences of outages in power distribution networks can be achieved by the installation of sectionalizing switches.
- Various objectives are used to optimize positions of switches, for example
  - the System Average Interruption Frequency Index (SAIFI),
  - the System Average Interruption Duration Index (SAIDI),
  - the Average Energy Not Supplied (AENS).
- Optimization of one of the objectives is often not equivalent to the minimization of another one.
- Research goal: Development and comparison of algorithms to simultaneously minimize several reliability factors in single-feeder distribution networks with a radial topology.

Assumptions and notations

- The distribution network with a tree structure:
  - $V = \{v_1, v_2, \ldots, v_m\}$ is the set of distribution and load nodes,
  - the supply node v_{m+1} is the root of the tree, $n = m + 1$,
  - load nodes are leaves,
  - c_j is the connection line between v_j and its parent,
  - there are m connection lines in the network.
- Distribution network parameters:
  - $N_j \geq 0$ is the number of users of the node v_j,
  - $P_j \geq 0$ is average active power of the node v_j.
  - λ_{v_j} and λ_{c_j} are the average failure rates of the node v_j and the line segment c_j,
  - t_{v_j} and t_{c_j} are the average total duration of failures during one year of v_j and c_j.

Objective functions: SAIFI, SAIDI, and AENS

- System Average Interruption Frequency Index (SAIFI),

$$\mathrm{SAIFI} = \frac{\sum_{j=1}^{m} \mu_j N_j}{\sum_{j=1}^{m} N_j},$$

  where μ_j is the average number of interruptions involving the node v_j during one year.

- System Average Interruption Duration Index (SAIDI)

$$\mathrm{SAIDI} = \frac{\sum_{j=1}^{m} U_j N_j}{\sum_{j=1}^{m} N_j},$$

  where U_j is the average total duration of all interruptions involving the node v_j during one year.

- Average Energy Not Supplied (AENS)

$$\mathrm{AENS} = \sum_{j=1}^{m} U_j P_j.$$

Multiobjective optimization problem

- The number of switches to be allocated is p.
- There are m admissible positions of switches (in each connection line).
- The search space: $X = \{Q \subset \{1, 2, \ldots, m\} : \#Q = p\}$.
- $\mathrm{SAIFI}(Q)$, $\mathrm{SAIDI}(Q)$, and $\mathrm{AENS}(Q)$ are the objectives for the case when switches are at positions in the set Q:
- A dominated solution is a solution, which is worse than another solution for each objective function.
- The set of non-dominated solutions is called the Pareto front: $X_P = \{Q \in X : Q \text{ is non-dominated}\}$.
- Multiobjective optimization problem: For given p find all non-dominated solutions with p sectionalizing switches.

Exhaustive search (ES)

- The exhaustive search method:
  - the Pareto front is initialized as $\Omega_{\mathrm{P}} = \emptyset$,
  - for each test solution $Q \in \Omega$, the values of objective functions $F_k(Q)$ are computed and the Pareto front is updated:
    - solutions belonging to Ω_{P} which are dominated by Q are removed from Ω_{P},
    - if Q is not dominated by any solution belonging to Ω_{P} then it is added to Ω_{P},
  - once all solutions $Q \in \Omega$ are considered, the set Ω_{P} is the complete Pareto front for the problem considered.
- Properties:
  - the algorithm is guaranteed to find the complete Pareto front,
  - for m line segments and p sectionalizing switches the number of test solutions is $N = \binom{m}{p}$.

Evolutionary algorithm (EA)

- The evolutionary algorithm:
  - The initial generation of size g_s is selected randomly.
  - In each of g_n steps a new generation is created using selection, crossover and mutation operations.
  - The domination-based tournament selection procedure is used to promote non-dominated individuals.
  - The number of runs is $r \geq 1$.
  - In each run a set of non-dominated solutions in the final population is found.
  - Non-dominated solutions found in all runs are used to construct the final result.
- Properties
  - The algorithm is heuristic. It is not guaranteed that the complete Pareto front is found.
  - The total number of test solutions is $N = rg_n g_s$, where g_s is the generation size and g_n is the number of generations.

Tree structure based (TS) algorithm

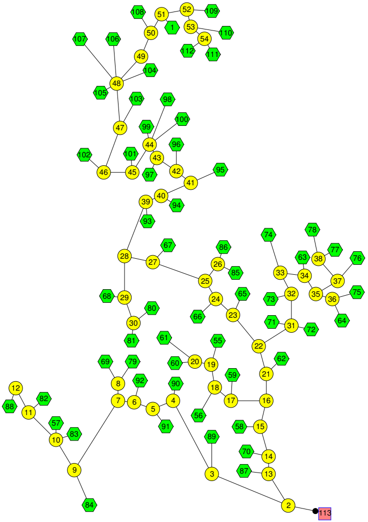
- The algorithm is based on the single-objective tree structure optimization algorithm.
- Tree structure based algorithm:
  - visit nodes following the tree structure from leaves to the root node,
  - at a given node:
    - construct partial solutions based on partial solutions found previously for its children,
    - for each partial solution compute gains in all objective functions obtained for this partial solution,
    - compare partial solutions and skip those which cannot lead to a non-dominated solution (to prevent from the exponential growth of the number of partial solutions),
  - at the root node from the set of complete solutions select non-dominated solutions.
- Properties:
  - the algorithm is guaranteed to find the complete Pareto front,
  - the number of partial solutions is very small when compared to the ES algorithm.

Single feeder test distribution networks

- Four distribution networks with $m = 76$, 112 (figure), 199, and 276 line segments.

- Failure rates:
  - 3.1 faults in one year for every 100 km of a line segment,
  - $\lambda_{c_j} = 3.1 \times 10^{-5} l_j$ for a line segment with the length l_j,
  - $\lambda_{v_j} = 0.03$ for user nodes,
  - $\lambda_{v_j} = 0.002$ for distribution nodes.

- Average fault durations:
  - $\tau_{c_j} = 0.983\,\mathrm{h}$ for line segments,
  - $\tau_{v_j} = 1\,\mathrm{h}$ for user nodes,
  - $\tau_{v_j} = 0.5\,\mathrm{h}$ for distribution nodes.



Performance of optimization algorithms, $m = 76$

$m = 76$ — the number of line segments, p — the number sectionalizing switches, S — the Pareto front size.

		TS		ES		EA		
p	S	N	t[s]	N	t[s]	N	t[s]	S'
1	**3**	237	0.00	76	0.00	1000	0.03	**3**
2	**3**	469	0.01	2850	0.03	4000	0.12	**3**
3	**2**	717	0.01	70300	0.57	36000	0.99	**2**
4	**1**	890	0.01	1282975	8.86	36000	1.00	**1**
5	**1**	1068	0.02	$1.8 \cdot 10^7$	127.43	36000	1.02	**1**
6	**2**	1296	0.02	$2.2 \cdot 10^8$	1733.44	400000	11.64	**2**
7	**2**	1531	0.02			400000	11.82	**2**
8	**4**	1725	0.04			$2.5 \cdot 10^6$	70.93	**4**
9	**8**	1909	0.04			$4 \cdot 10^7$	1074.16	**8**
10	**6**	2084	0.06			$9 \cdot 10^7$	2550.01	**6**
11	**4**	2210	0.07			$4 \cdot 10^7$	1085.21	**4**
12	**6**	2347	0.08			$9 \cdot 10^7$	2588.74	4
13	**4**	2536	0.10			$9 \cdot 10^7$	2603.86	**4**
14	**4**	2714	0.08			$9 \cdot 10^7$	2626.83	3
15	**6**	2860	0.11			$9 \cdot 10^7$	2650.72	1

Performance of optimization algorithms, $m = 112$

$m = 112$ — the number of line segments, p — the number sectionalizing switches, S — the Pareto front size.

		TS		ES		EA		
p	S	N	t[s]	N	t[s]	N	t[s]	S'
1	**2**	434	0.01	112	0.00	1000	0.05	**2**
2	**2**	1047	0.01	6216	0.07	4000	0.22	**2**
3	**1**	1786	0.03	227920	2.10	25000	1.29	**1**
4	**1**	2376	0.04	$6.2 \cdot 10^6$	56.66	36000	1.76	**1**
5	**2**	2846	0.05	$1.3 \cdot 10^8$	1357.23	900000	52.37	**2**
6	**4**	3225	0.07			$1.6 \cdot 10^6$	85.68	**4**
7	**2**	3572	0.10			$2.5 \cdot 10^6$	127.38	**2**
8	**2**	4006	0.13			$4 \cdot 10^7$	1958.18	**2**
9	**7**	4606	0.17			$9 \cdot 10^7$	4771.18	**7**
10	**11**	5335	0.21			$9 \cdot 10^7$	4800.77	**11**
11	**16**	6068	0.25			$9 \cdot 10^7$	4790.01	**16**
12	**12**	6696	0.32			$9 \cdot 10^7$	4773.47	10
13	**11**	7173	0.41			$9 \cdot 10^7$	4694.34	9
14	**5**	7600	0.47			$9 \cdot 10^7$	4703.44	2
15	**8**	8058	0.52			$9 \cdot 10^7$	4736.94	3

Performance of optimization algorithms, $m = 199$

$m = 199$ — the number of line segments, p — the number sectionalizing switches, S — the Pareto front size.

		TS		ES		EA		
p	S	N	t[s]	N	t[s]	N	t[s]	S'
1	**2**	767	0.02	199	0.01	4000	0.21	**2**
2	**6**	2049	0.03	19701	0.42	16000	0.85	**6**
3	**6**	5113	0.09	$1.3 \cdot 10^6$	26.36	100000	5.05	**6**
4	**7**	14355	0.28	$6.3 \cdot 10^7$	1384.54	225000	11.63	**7**
5	**5**	41294	0.87			$4.9 \cdot 10^6$	256.02	**5**
6	**5**	106112	2.88			$4 \cdot 10^7$	1796.57	**5**
7	**5**	237598	9.16			$9 \cdot 10^7$	4180.05	**5**
8	**5**	472232	23.02			$9 \cdot 10^7$	4198.28	4
9	**6**	850364	44.71			$9 \cdot 10^7$	4237.13	3
10	**8**	$1.4 \cdot 10^6$	63.62			$9 \cdot 10^7$	4228.11	1
11	**10**	$2.2 \cdot 10^6$	106.10			$9 \cdot 10^7$	4163.26	0
12	**9**	$3.2 \cdot 10^6$	153.21			$9 \cdot 10^7$	4186.69	1
13	**8**	$4.4 \cdot 10^6$	198.38			$9 \cdot 10^7$	4207.85	0
14	**6**	$5.9 \cdot 10^6$	243.60			$9 \cdot 10^7$	4223.61	0
15	**4**	$7.7 \cdot 10^6$	315.71			$9 \cdot 10^7$	4152.89	0

Performance of optimization algorithms, $m = 262$

$m = 262$ — the number of line segments, p — the number sectionalizing switches, S — the Pareto front size.

		TS		ES		EA		
p	S	N	t[s]	N	t[s]	N	t[s]	S'
1	**1**	972	0.01	262	0.01	4000	0.27	**1**
2	**1**	2764	0.04	34191	0.64	9000	0.56	**1**
3	**1**	9258	0.11	$3 \cdot 10^6$	54.95	64000	4.08	**1**
4	**1**	39671	0.45	$2 \cdot 10^8$	3566.35	144000	8.78	**1**
5	**5**	$1.6 \cdot 10^5$	3.01			$6 \cdot 10^6$	400.10	**5**
6	**7**	$5.7 \cdot 10^5$	14.97			$4 \cdot 10^7$	2456.41	**7**
7	**11**	$1.7 \cdot 10^6$	53.79			$9 \cdot 10^7$	5142.64	**11**
8	**11**	$4.2 \cdot 10^6$	142.37			$9 \cdot 10^7$	5158.91	10
9	**8**	$9.1 \cdot 10^6$	360.82			$9 \cdot 10^7$	5117.55	2
10	**10**	$1.8 \cdot 10^7$	1071.48			$9 \cdot 10^7$	5090.11	0
11	**8**	$3.4 \cdot 10^7$	2393.80			$9 \cdot 10^7$	5106.81	0
12	**8**	$5.7 \cdot 10^7$	4726.02			$9 \cdot 10^7$	5105.97	0

Pareto front example: switch locations

- $m = 112$ line segments,
- $p = 10$ sectionalizing switches;
- design objectives: AENS, SAIDI and SAIFI.

AENS	SAIDI	SAIFI	Switch locations
3964.18	0.878326	0.901420	3,7,13,17,23,29,31,39,45,49
3897.36	0.890346	0.914226	3,6,13,17,23,26,29,31,41,48
3910.84	0.886511	0.910306	3,7,13,17,23,26,29,31,41,48
3913.29	0.885074	0.908474	3,6,13,17,23,26,29,31,41,49
3926.76	0.881239	0.904554	3,7,13,17,23,26,29,31,41,49
3964.64	0.874940	0.899122	3,13,17,23,26,29,31,41,48,87
3980.56	0.869667	0.893370	3,13,17,23,26,29,31,41,49,87
3997.84	0.864212	0.887258	3,13,17,23,26,29,31,39,45,49
3984.77	0.867688	0.890851	3,13,17,23,26,29,31,40,45,49
3976.33	0.872723	0.896036	3,13,17,23,26,29,31,40,46,49
3972.27	0.874575	0.898877	3,14,17,23,26,29,31,41,48,87

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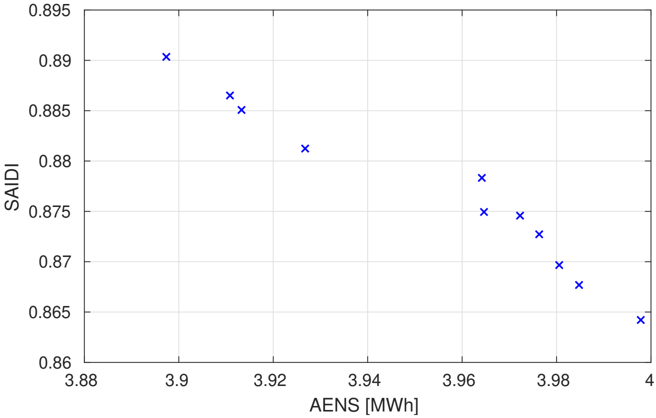

Conclusions

- Three approaches to solve the multiobjective optimization algorithm for switch allocation in radial power distribution grids have been compared.
  - The tree search based algorithm and the exhaustive search approach are guaranteed to find the complete Pareto front.
  - The tree search based algorithm is much faster than other methods.
  - The evolutionary algorithm is faster than the exhaustive search method. It is heuristic and is not guaranteed to find the complete Pareto front.
- The results of multiobjective optimization show that usually a single solution cannot be considered optimal from the point of view of AENS/SAIDI/SAIFI trade-offs in switch placement problems.
- Finding the complete set of non-dominated solutions may be helpful in the design of power distribution networks.