

# On the modeling of blackouts in power networks

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# Modeling of blackouts in power networks

- A *power network* is a set of functionally related devices that are electrically connected for the sake of transmitting and distributing electricity in a specified area.
- *Power blackout* (or power outage) is a loss of electric power in a given area (a large portion of the network).
- We propose a model of power distribution system which may be effective for modeling of power blackouts:
- The model is a combination of
  - the admittance model, and
  - a probability model of faults of network components.

# The admittance model of power networks

- The network is composed of  $N$  nodes connected by  $M$  transmission lines,
- Four types of nodes:
  - *consumer nodes* where energy is dissipated,
  - *distribution nodes* which transmit energy,
  - *generation nodes* which supply of energy to the network,
  - *transformer nodes* where high-voltage, mid-voltage and low-voltage grids are connected.
- Transmission lines:
  - $Y_{kj}$  is the admittance of the transmission line connecting nodes  $k$  and  $j$ ,
  - $Y_{kj} = 0$  if the nodes  $k$  and  $j$  are not connected
- The analysis is carried out in “per unit” (p.u.).
  - transformer voltages at both sides are scaled to base values,
  - hence, there is no difference between transformer nodes and distribution nodes from the admittance model point of view.

# The admittance model of power networks, cont.

- The current Kirchhoff's law for transformer or a distribution node  $k$  yields

$$\sum_{j=1, j \neq k}^N Y_{jk}(V_k - V_j) = 0.$$

- Consumer nodes differ by a non-zero current flowing from the node to the load:

$$\sum_{j=1, j \neq k}^N Y_{jm}(V_k - V_j) = -I_k,$$

- Generation nodes provide fixed voltages

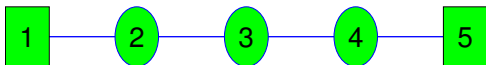
$$V_k = v_k,$$

- The whole network can be described using the tableau formulation

$$AV = b.$$

# The admittance model: example

- Power network composed of two generation nodes (rectangles) and three consumer nodes (circles):



- The tableau formulation of the admittance model:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -Y_{12} & Y_{12} + Y_{23} & -Y_{23} & 0 & 0 \\ 0 & -Y_{23} & Y_{23} + Y_{34} & -Y_{34} & 0 \\ 0 & 0 & -Y_{34} & Y_{34} + Y_{45} & -Y_{45} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{pmatrix} = \begin{pmatrix} v_1 \\ -I_2 \\ -I_3 \\ -I_4 \\ v_5 \end{pmatrix}$$

# Modeling of power network failures

- Each component of the network is characterized by its maximum capacity:
  - For the  $m$ th transmission line the *current capacity*  $I_{m,\max}$  is the maximum current which can be transmitted through this line.
  - We say that the node  $k$  exceeds its power capacity  $P_{k,\max}$  if  $\sum_j |V_k I_{jk}| > P_k$ , where  $I_{jk} = Y_{jk}(V_j - V_k)$  is the current between nodes  $j$  and  $k$ .
- Failure of any component of the network (or simultaneous failures of several components) may be a cause of a cascading failure (capacities of other components of the network are exceeded).
- Power network reliability assessment:
  - The *percentage of unserved consumer nodes* (PUCN) — the percentage of consumer nodes for which the supplied voltage after a given failure falls below the minimum value.
  - The *percentage of unserved consumer power* (PUCP) — the percentage of a drop in the total consumer power due to a given failure.

# The probabilistic model of failures

- $S = \{1, 2, \dots, N + M\}$  — the set of all elements of the network.
- $p_k$  — the probability that the  $k$ th element fails in the time unit ( $p_k$  can be estimated based on history of past failures, age of a given element, weather forecast, etc.).
- For a subset  $T \subset S$  by  $P(T)$  we denote the probability that elements in the set  $T$  fail in a given time unit, while the other elements do not fail.
- $PUCN(T)$  and  $PUCP(T)$  the percentage of unserved consumer nodes and the percentage of unserved consumer power induced by faults of elements in the set  $T$ .
- $PUCN_{BLACKOUT}$ ,  $PUCP_{BLACKOUT}$  — thresholds characterizing a blackout (an event affecting a large portion of the network).
- The probability that a blackout occurs in a time unit

$$P_{BLACKOUT} = \sum_{T \subset S: PUCN(T) \geq PUCN_{BLACKOUT}} P(T).$$

# Computing the probability of a blackout

- Considering only single-failure events (assumption: the chance that two or more independent failures occur in a given time unit is negligible):

$$P_{\text{BLACKOUT}} \approx \sum_{k \in S: \text{PUCN}(\{k\}) \geq \text{PUCN}_{\text{BLACKOUT}}} P(\{k\}).$$

- Considering also double-failure events:

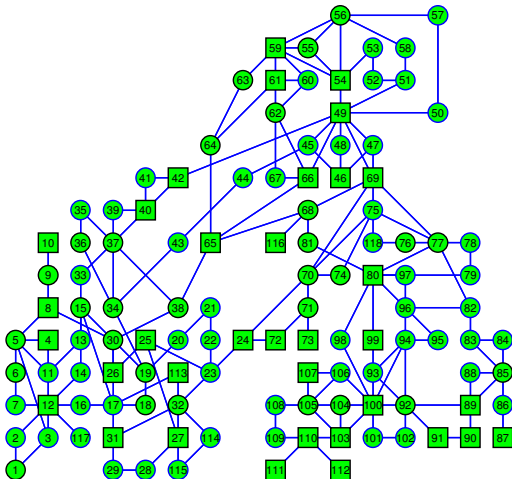
$$P_{\text{BLACKOUT}} \approx \sum_{T \subset S, \#T \leq 2: \text{PUCN}(T) \geq \text{PUCN}_{\text{BLACKOUT}}} P(T).$$

- Approximating  $P(T)$ :
  - If  $p_k$  is small then  $P(\{k\}) \approx p_k$ .
  - If additionally individual failures are independent then  $P(\{k, l\}) \approx p_k p_l$   
(not necessarily true, for example for weather induced events).



# Example test network: the IEEE 118 bus

- 118 nodes and 186 transmission lines; generation nodes — rectangles, consumer nodes — blue circles, transformer and distribution nodes — black circles:

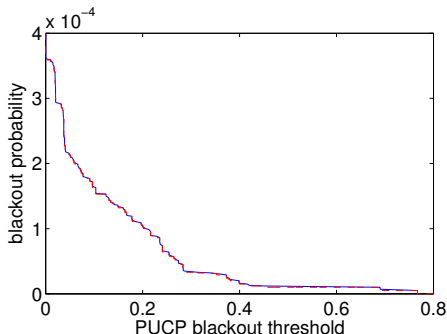
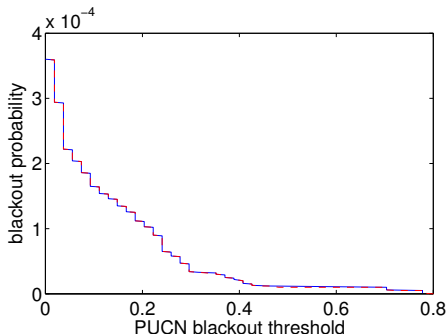


# Test conditions

- Generator nodes: the voltage is  $v = 1$  (analysis is carried out in “per unit” (p.u.)).
- Consumer nodes: the current of each load is  $I_k = 1$ .
- The admittance of each connection is  $Y_{kl} = 11$ .
- Nominal values of transmission lines currents and powers of nodes are assumed to be equal to the values existing under the conditions presented above.
- Capacities of network elements are assumed to be 20% larger than nominal values.
- The probability that a given node or transmission line fails in the time unit is equal to  $10^{-6}$  and  $2 \times 10^{-6}$ , respectively.

# Blackout probability versus blackout threshold

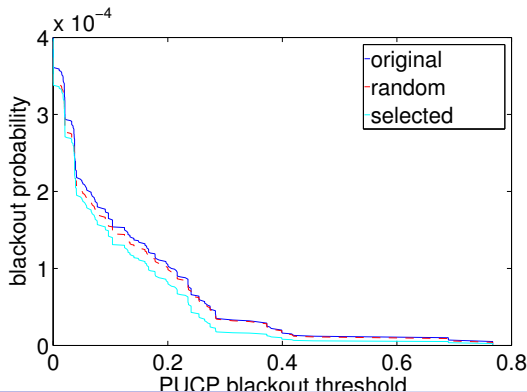
- Blackout probability for single-failure events only (solid line), and for single- and double-failure events (dashed line):



- Conclusion: errors caused by not considering double-failure events can be neglected for the considered failure probabilities and the network size.

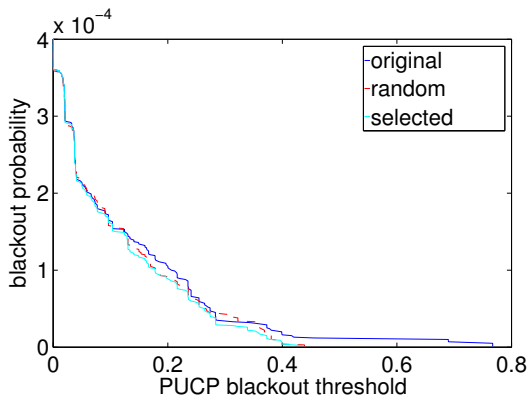
# Improving susceptibility to power blackouts

- Reduce by half failure probabilities of 10% elements of the network.
- Blackout probability for the original network (blue); and for the improved network: elements selected randomly (dashed red); elements selected according to the largest cascading failure criterion (cyan):



# Improving susceptibility to power blackouts

- Doubling capacities of 10% of elements of the network.
- Blackout probability for the original network (blue); and for the improved network: elements selected randomly (dashed red); elements selected according to the largest number of capacity violation criterion (cyan)



# Conclusions

- We have described a combined admittance and probabilistic model of power networks useful for computing the probability of a blackout.
- The model has been used to characterize performance of an example power network.
- It has been shown that the model can be used to predict power outages and to develop measures to reduce their risk.
- In future, we plan to apply this model to the analysis of existing networks using real data provided by electric power companies with the goal of developing suggestions to improve existing power networks.