Tree Structure Based Algorithm for Multiobjective Optimization of Switch Allocation in Radial Distribution Networks

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Introduction

- Reducing the consequences of outages in power distribution networks can be achieved by the installation of **sectionalizing switches** [1].
- Various objectives are used to optimize positions of switches, for example
  - the System Average Interruption Frequency Index (SAIFI),
  - the System Average Interruption Duration Index (SAIDI),
  - the Average Energy Not Supplied (AENS).
- Optimization of one of the objectives is often not equivalent to the minimization of another one [2].

Research goal

Development of a fast algorithm to simultaneously minimize several reliability factors in single-feeder distribution networks with a radial topology.

Assumptions and notations

- The distribution network contains m distribution and load nodes v_j, $1 \le j \le m$ and a single generator node v_{m+1}, $n = m+1$.
- The network has a tree structure: the generator node is the root of the tree, load nodes are leaves.
- c_j is the connection line between v_j with its parent.
- Failure rates of v_j and c_j are λ_{v_j}, λ_{c_j}; $\lambda_j = \lambda_{v_j} + \lambda_{c_j}$.
- Total durations of failures in one year of v_j and c_j are t_{v_j}, t_{c_j}; $t_j = t_{v_j} + t_{c_j}$.
- $N_j \ge 0$ and $P_j \ge 0$ are the number of users and the average (active) power of the node v_j.
- C_j and D_j are the sets of indexes of children and descendants of v_j, respectively.
- $\bar{P}_j = P_j + \sum_{i \in D_j} P_i$, $\bar{N}_j = N_j + \sum_{i \in D_j} N_i$, $\bar{t}_j = t_j + \sum_{i \in D_j} t_i$, $\bar{\lambda}_j = \lambda_j + \sum_{i \in D_j} \lambda_i$.

Multiobjective optimization

- The number of switches to be allocated is p.
- There are m admissible positions of switches.
- The search space: $X = \{Q \subset \{1,\ldots,m\} \colon \#Q = p\}$.
- R_j is the set of switches in $Q_j = Q \cap D_j$ with a path from v_j not passing through another switch.
- SAIFI(Q), SAIDI(Q), and AENS(Q) are the objective functions for the case when switches are at positions in the set $Q \subset \{1,2,\ldots,m\}$:

$$\mathrm{AENS}(Q) = \bar{P}_n \cdot \bar{t}_n - \sum_{j \in Q} \left(\bar{P}_n - \bar{P}_j\right)\left(\bar{t}_j - \sum_{i \in R_j} \bar{t}_i\right),$$

$$\mathrm{SAIDI}(Q) = \bar{t}_n - \sum_{j \in Q} \frac{\bar{N}_n - \bar{N}_j}{\bar{N}_n}\left(\bar{t}_j - \sum_{i \in R_j} \bar{t}_i\right),$$

$$\mathrm{SAIFI}(Q) = \bar{\lambda}_n - \sum_{j \in Q} \frac{\bar{N}_n - \bar{N}_j}{\bar{N}_n}\left(\bar{\lambda}_j - \sum_{i \in R_j} \bar{\lambda}_i\right).$$

- A **dominated solution** is a solution, which is worse than another solution for each objective.
- The set of non-dominated solutions is called the **Pareto front**: $X_P = \{Q \in X \colon Q \text{ is non-dominated}\}$.

Problem formulation

For given p solve the multiobjective optimization problem, i.e., **find all non-dominated solutions with p sectionalizing switches**.

Tree structure based algorithm

- The algorithm is based on the single-objective tree structure optimization algorithm [3].
- Outline of the algorithm:
  - Visit nodes following the tree structure from leaves to the root node and construct partial solutions.
  - Partial solutions for a given node are constructed based on partial solutions found previously for its children.
  - The partial solution s generated by Q at the position j is the set of switches $Q_s = Q \cap (\{j\} \cup D_j)$.
  - For each partial solution compute gains in all objective functions obtained for this partial solution:

$$g_{\mathrm{AENS}} = \sum_{k \in Q_s} \left(\bar{P}_n - \bar{P}_k\right)\left(\bar{t}_k - \sum_{i \in R_k} \bar{t}_i\right),$$

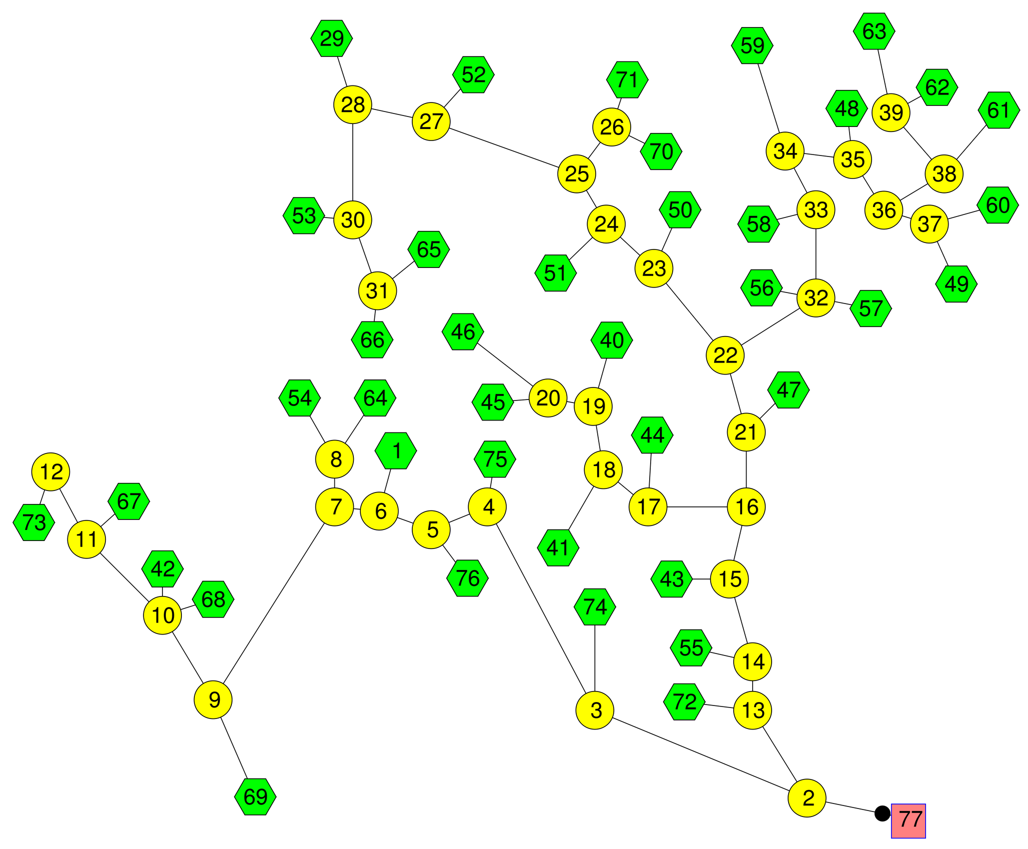
$$g_{\mathrm{SAIDI}} = \sum_{k \in Q_s} \frac{\bar{N}_n - \bar{N}_k}{\bar{N}_n}\left(\bar{t}_k - \sum_{i \in R_k} \bar{t}_i\right),$$

$$g_{\mathrm{SAIFI}} = \sum_{k \in Q_s} \frac{\bar{N}_n - \bar{N}_k}{\bar{N}_n}\left(\bar{\lambda}_k - \sum_{i \in R_k} \bar{\lambda}_i\right).$$

  - At each node compare partial solutions and skip those which cannot lead to a non-dominated solution (to prevent from the exponential growth of the number of partial solutions).
  - At the root node from the set of complete solutions select non-dominated solutions.

Test distribution network

- An existing network with $m = 76$ line segments, 39 load nodes, 37 distribution nodes and a single supply node.



Computational complexity

- Comparison of the tree structure (TS) based algorithm with the exhaustive search (ES) method.
- Computation times (in seconds, 3.4 GHz processor) to find the complete Pareto front:

p	ES	TS	p	ES	TS
1	0.00	0.00	8		0.04
2	0.04	0.01	9		0.05
3	0.57	0.01	10		0.07
4	8.70	0.01	11		0.07
5	124.94	0.01	12		0.08
6	1706.25	0.02	13		0.10
7		0.02	14		0.08

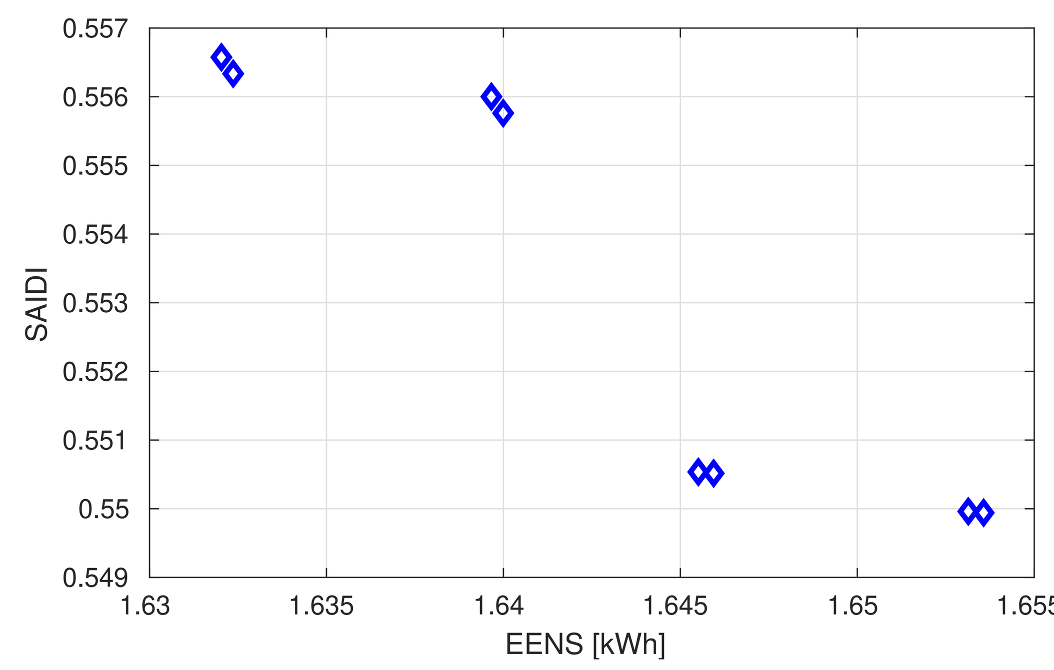
- The ES method works only for $p \le 6$ (the computation time grows exponentially with p).
- The results obtained using both methods are the same (for $p \le 6$).
- Finding all non-dominating solutions for $p \le 14$ using the TS algorithm takes a fraction of a second.
- **Conclusion:** the proposed multiobjective optimization algorithm is very efficient.

Optimization results

- Pareto front sizes for various combinations of objective functions:

p	AENS SAIDI	AENS SAIFI	SAIDI SAIFI	AENS SAIDI SAIFI
1	3	3	1	3
2	3	3	1	3
3	2	2	1	2
4	1	1	1	1
5	1	1	1	1
6	2	2	1	2
7	2	2	1	2
8	4	4	1	4
9	8	6	2	8
10	6	3	2	6
11	4	1	4	4
12	6	3	2	6
13	4	2	2	4
14	4	2	2	4
15	6	3	2	6

- In most cases the Pareto front is nontrivial—contains more than one element (neither solution can be considered to be optimal for all objective functions simultaneously).
- Example: the Pareto front for the AENS/SAIDI multiobjective optimization problem, $p = 9$:



Conclusions

- A tree structure based algorithm to solve the switch allocation problem with multiple objectives has been proposed.
- The algorithm has been tested using a network of a moderate size showing its high efficiency.
- Multiobjective optimization may by useful in the design and modernization of distribution networks.

References

[1] R. Billinton and S. Jonnavithula. Optimal switching device placement in radial distribution systems. *IEEE Trans. Power Delivery*, 11(3):1646–1651, 1996.

[2] J. R. Bezerra, G. C. Barroso, R. P. S. Leao, and R. F. Sampaio. Multiobjective optimization algorithm for switch placement in radial power distribution networks. *IEEE Transactions on Power Delivery*, 30(2):545–552, 2015.

[3] Z. Galias. On optimum placement of sectionalizing switches in radial distribution networks. In *Proc. IEEE Int. Symp. Circuits Syst. (ISCAS)*, pp. 1030–1033, Baltimore, May 2017.

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