Tree Structure Based Algorithm for Multiobjective Optimization of Switch Allocation in Radial Distribution Networks

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Research goal

- Reducing the consequences of outages in power distribution networks can be achieved by the installation of sectionalizing switches.
- Various objectives are used to optimize positions of switches, for example
  - the System Average Interruption Frequency Index (SAIFI),
  - the System Average Interruption Duration Index (SAIDI),
  - the Average Energy Not Supplied (AENS).
- Optimization of one of the objectives is often not equivalent to the minimization of another one.
- Goal: Development of a fast algorithm to simultaneously minimize several reliability factors in single-feeder distribution networks with a radial topology.

Assumptions and notations

- The distribution network with a tree structure:
  - $V = \{v_1, v_2, \ldots, v_m\}$ is the set of distribution and load nodes,
  - the supply node v_{m+1} is the root of the tree, $n = m + 1$,
  - load nodes are leaves,
  - c_j is the connection line between v_j and its parent,
  - C_j is the set of indexes of children of v_j.
  - D_j is the set of indexes of descendants of v_j.
- Distribution network parameters:
  - $N_j \geq 0$ is the number of users of the node v_j,
  - $P_j \geq 0$ is average active power of the node v_j.
  - accumulated values: $\bar{P}_j = P_j + \sum_{i \in D_j} P_i$, $\bar{N}_j = N_j + \sum_{i \in D_j} N_i$,
  - λ_{v_j} and λ_{c_j} are the average failure rates of the node v_j and the line segment c_j,
  - t_{v_j} and t_{c_j} are the average total duration of failures during one year v_j and c_j,
  - $\lambda_j = \lambda_{v_j} + \lambda_{c_j}$, $t_j = t_{v_j} + t_{c_j}$,
  - accumulated values: $\bar{t}_j = t_j + \sum_{i \in D_j} t_i$, $\bar{\lambda}_j = \lambda_j + \sum_{i \in D_j} \lambda_i$.

Objective functions: SAIFI, SAIDI, and AENS

- System Average Interruption Frequency Index (SAIFI),

$$\mathrm{SAIFI} = \frac{\sum_{j=1}^{m} \mu_j N_j}{\sum_{j=1}^{m} N_j},$$

μ_j is the average number of interruptions.

- System Average Interruption Duration Index (SAIDI)

$$\mathrm{SAIDI} = \frac{\sum_{j=1}^{m} U_j N_j}{\sum_{j=1}^{m} N_j},$$

U_j is the average total duration of all interruptions involving the node v_j during one year.

- Average Energy Not Supplied (AENS)

$$\mathrm{AENS} = \sum_{j=1}^{m} U_j P_j.$$

Multiobjective optimization problem

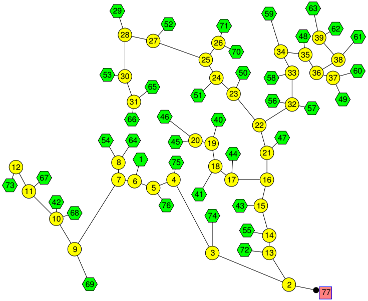
- The number of switches to be allocated is p.
- There are m admissible positions of switches.
- The search space: $X = \{Q \subset \{1, 2, \ldots, m\} \colon \#Q = p\}$.
- R_j is the set of switches in $Q_j = Q \cap D_j$ with a path from v_j not passing through another switch.
- $\mathrm{SAIFI}(Q)$, $\mathrm{SAIDI}(Q)$, and $\mathrm{AENS}(Q)$ are the objectives for the case when switches are at positions in the set Q:
- A dominated solution is a solution, which is worse than another solution for each objective function.
- The set of non-dominated solutions is called the Pareto front: $X_P = \{Q \in X \colon Q \text{ is non-dominated}\}$.
- Multiobjective optimization problem: For given p find all non-dominated solutions with p sectionalizing switches.

Tree structure based algorithm

- The algorithm is based on the single-objective tree structure optimization algorithm.
- Outline of the algorithm:
  - Visit nodes following the tree structure from leaves to the root node and construct partial solutions.
  - Partial solutions for a given node are constructed based on partial solutions found previously for its children.
  - The partial solution s generated by Q at the position j is the set of switches $Q_s = Q \cap (\{j\} \cup D_j)$.
  - For each partial solution compute gains in all objective functions obtained for this partial solution.
  - At each node compare partial solutions and skip those which cannot lead to a non-dominated solution (to prevent from the exponential growth of the number of partial solutions).
  - At the root node from the set of complete solutions select non-dominated solutions.

Test distribution network

- A real network with from the southern part of Poland:
  - $m = 76$ line segments,
  - 39 load nodes,
  - 37 distribution nodes,
  - a single supply node.



- Failure rates (data provided by the electricity company):
  - 3.1 faults in one year for every 100 km of a line segment,
  - $\lambda_{c_j} = 3.1 \times 10^{-5} l_j$ for a line segment with the length l_j,
  - $\lambda_{v_j} = 0.03$ for user nodes,
  - $\lambda_{v_j} = 0.002$ for distribution nodes.
- Average fault durations:
  - $\tau_{c_j} = 0.983\,\mathrm{h}$ for line segments,
  - $\tau_{v_j} = 1\,\mathrm{h}$ for user nodes,
  - $\tau_{v_j} = 0.5\,\mathrm{h}$ for distribution nodes.

Computational complexity

- Computation times (in seconds, 3.4 GHz processor) to find the complete Pareto front using the the tree structure based algorithm (TS) and the exhaustive search method (ES).

p	ES	TS
1	0.00	0.00
2	0.04	0.01
3	0.57	0.01
4	8.70	0.01
5	124.94	0.01
6	1706.25	0.02
7		0.02
8		0.04
9		0.05
10		0.07
11		0.07
12		0.08
13		0.10
14		0.08

- The ES method works only for $p \leq 6$ (the computation time grows exponentially with p).
- The results obtained using both methods are the same (for $p \leq 6$).
- Finding all non-dominating solutions for $p \leq 14$ using the TS algorithm takes a fraction of a second.

- Conclusion: the proposed multiobjective optimization algorithm is very efficient.

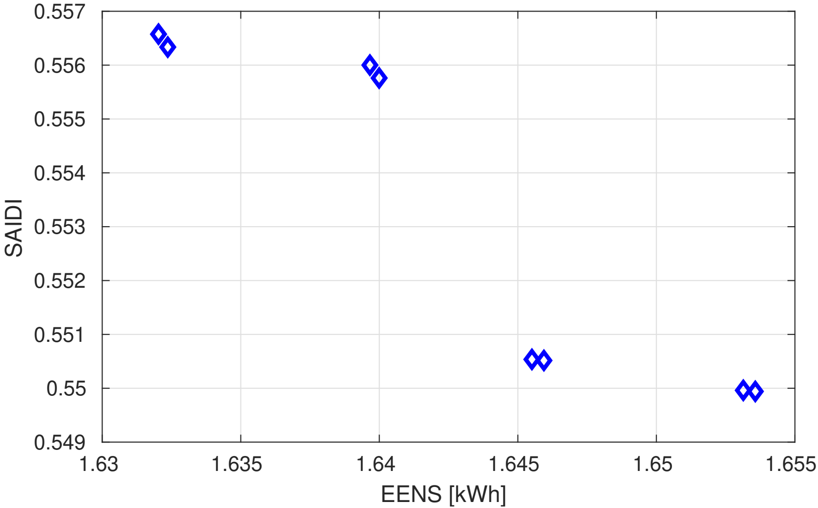
Optimization results

- Pareto front sizes for various combinations of objective functions:

p	AENS SAIDI	AENS SAIFI	SAIDI SAIFI	AENS SAIDI SAIFI
1	3	3	1	3
2	3	3	1	3
3	2	2	1	2
4	1	1	1	1
5	1	1	1	1
6	2	2	1	2
7	2	2	1	2
8	4	4	1	4
9	8	6	2	8
10	6	3	2	6
11	4	1	4	4
12	6	3	2	6
13	4	2	2	4
14	4	2	2	4

- In most cases the Pareto front is nontrivial—contains more than one element (neither solution can be considered to be optimal for all objective functions simultaneously).

Example: the Pareto front for the AENS/SAIDI multiobjective optimization problem, $p = 9$



Conclusions

- A tree structure based algorithm to solve the switch allocation problem with multiple objectives has been proposed.
- The algorithm has been tested using a network of a moderate size showing its high efficiency.
- Multiobjective optimization may by useful in the design and modernization of distribution networks.