# QUADRATURE CHAOS SHIFT KEYING

*Zbigniew Galias* * *and* *Gian Mario Maggio* †

Institute for Nonlinear Science, University of California, San Diego
9500 Gilman Drive, La Jolla, CA 92093–0402, U.S.A.

**ABSTRACT**

In this paper we propose a multilevel version of the differential chaos shift keying (DCSK) scheme exhibiting the same bit error rate performance as DCSK but double data rate and spectral efficiency.

## 1. INTRODUCTION

In the last few years a great research effort has been devoted towards the development of efficient chaos-based modulation techniques [2, 5]. Among the several schemes proposed, one of the best bit error rate (BER) performances has been achieved by the DCSK system [5]. DCSK is a transmitted-reference digital signaling scheme. In DCSK a chaotic sample function is transmitted for half the symbol time followed, in the second half, by its duplicate or an inverted copy depending on the binary symbol ("0" or "1").

Recently, several different methods have been proposed to increase the bit rate of DCSK (see [3]). The simplest option consists of scaling the information and/or the reference parts of the signal. For example the information bearing part may be multiplied by a number depending on the symbol transmitted. A more sophisticated approach uses two basis chaotic functions and divides the symbol period into four time slots in order to obtain a multilevel scheme [6]. The methods, though, achieve higher data rate by giving up some of the BER performance.

In this work we introduce a new multilevel chaos-based communication scheme, that we call quadrature chaos shift keying (QCSK), with the same BER performance as DCSK but characterized by higher data rate.

## 2. DIFFERENTIAL CHAOS SHIFT KEYING

In DCSK two chaotic sample functions are sent for each symbol period, corresponding to one bit of information. The first sample function is used as a reference while the second represents the information to be transmitted. The latter is a copy or inverted copy of the first function for the transmitted symbol "0" and "1", respectively. On the receiver side one observes a noisy version of the transmitted signal. The digital information is extracted by means of a correlation process between the two received sample functions.

### 2.1. DCSK versus BPSK

We emphasize that DCSK is in some sense similar to the BPSK (binary phase shift keying) modulation scheme [1]. In BPSK one transmits a $\sin(\cdot)$ function signal or its inverted version depending on the bit of information. In principle DCSK does exactly the same except that the (chaotic) signal used for sending the information is different for each bit, thus one needs to send the corresponding reference signal as well in order to enable the detection at the receiver.

One of the modifications of BPSK is the QPSK (quadrature phase shift keying) scheme, which exhibits the same BER performance as BPSK, but is more efficient by having a double data rate. Basically, in QPSK a two-bit symbol is encoded as a linear combination of two orthogonal waveforms ($\sin$ and $\cos$). In the next section we describe how this idea can be applied for increasing the data rate of DCSK.

## 3. QUADRATURE CHAOS SHIFT KEYING

The aim of this section is to illustrate the theory behind the quadrature chaos shift keying communication system. The block diagram of the QCSK modulation scheme is shown in Fig. 1.

### 3.1. Orthogonal Chaotic Signals

The first step for introducing QCSK is the generation of a signal orthogonal to a chaotic reference signal. Let $x(t)$ be a chaotic reference signal defined for $t \in [0, \tau]$. Let us assume that the signal $x$ has zero mean value[1] and that in the

* galias@ucsd.edu. Permanent address: Dept. of Electrical Engineering, University of Mining and Metallurgy, al. Mickiewicza 30, 30–059 Kraków, Poland, galias@zet.agh.edu.pl.

† gmaggio@ucsd.edu. Corporate affiliation with STMicroelectronics, AST-La Jolla, e-mail: gian-mario.maggio@st.com.

This work was supported in part by ARO, grant No. DAAG 55–98–1–0269.

[1] This assumption simply implies that the DC value of the reference signal $x(t)$ is filtered out.

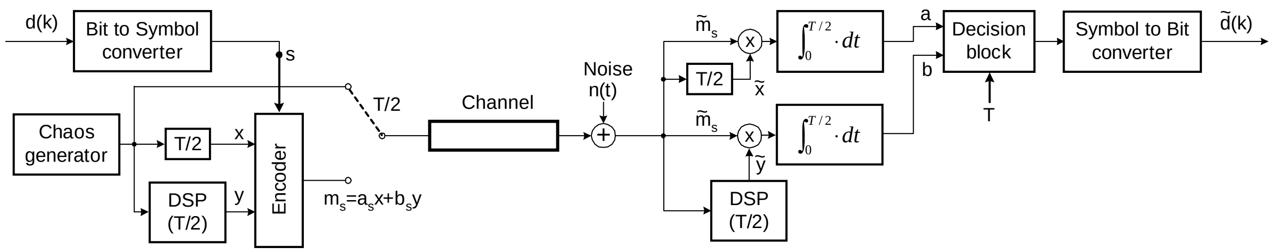


Figure 1: Block diagram of the QCSK scheme. The DSP unit producing the orthogonal signal introduces a delay equal to $T/2$.

interval $[0,\tau]$ it admits the the following Fourier expansion (with $f_0=0$):

$$x(t)=\sum_{k=1}^{\infty} f_k \sin(k\omega t+\varphi_k), \tag{1}$$

where $\omega = 2\pi/\tau$. Correspondingly, we denote by $P_x$ the average power of $x$ in the time interval $[0,\tau]$:

$$P_x = \frac{1}{\tau}\int_0^{\tau} x^2(t)dt = \frac{1}{2}\sum_{k=1}^{\infty} f_k^2.$$

Let's define the *complementary signal* $y(t)$, with $t\in[0,\tau]$, as the signal obtained by changing the phase of each Fourier frequency component by $\pi/2$:

$$y(t)=\sum_{k=1}^{\infty} f_k \sin(k\omega t+\varphi_k-\pi/2). \tag{2}$$

The signals $x(t)$ and $y(t)$ are orthogonal in the interval $[0,\tau]$ and have the same power, that is:

$$x\perp y \iff \frac{1}{\tau}\int_0^{\tau} x(t)y(t)dt = 0, \tag{3}$$

$$P_x = P_y \iff \frac{1}{\tau}\int_0^{\tau} x^2(t)dt = \frac{1}{\tau}\int_0^{\tau} y^2(t)dt. \tag{4}$$

The above properties follow from:

$$\frac{1}{\tau}\int_0^{\tau} f_k \sin(k\omega t+\phi_k-\alpha) f_m \sin(m\omega t+\phi_m-\beta)dt =$$
$$= \begin{cases} \frac{1}{2}f_k^2\cos(\alpha+\beta) & \text{for } k=m,\\ 0 & \text{for } k\neq m. \end{cases} \tag{5}$$

Referring to the definition (2) of $y$, we point out that by extending $x(t)$ and $y(t)$ to periodic signals with period $\tau$, $y(t)$ represents the Hilbert transform of $x(t)$. We recall that the Hilbert transform of a real signal is obtained by introducing a $\pi/2$ phase shift in every frequency component [1]. This property is well known and exploited for example in the context of amplitude modulation (AM) for obtaining a single (suppressed carrier) sideband (SSB) signal [1].

If we consider a length-$L$ chaotic sequence $(x_k)_{k=0}^{L-1}$ we can generate the orthogonal signal $(y_k)_{k=0}^{L-1}$ with the following procedure, in analogy with the AM-SSB modulation. Take the input sequence, subtract its mean value, calculate the fast Fourier transform (FFT), zero the coefficients corresponding to negative frequencies and then take the inverse FFT. In practice this can be achieved by means of a DSP (digital signal processor) unit as indicated in the block diagram of Fig. 1.

### 3.2. Chaotic Signal Constellations

Once we have an orthogonal basis of signals $(x,y)$ we can produce many different multilevel signaling schemes. In particular, we represent each symbol $s$ in the complex plane as

$$c_s = a_s + ib_s,$$

to which we associate the modulated signal:

$$m_s(t) = a_s x(t) + b_s y(t)$$

In this work we consider the four signal constellations shown in Fig. 2, which analytical representation is reported in Table 1. Constellations (a) and (b) are two-level signaling, the first one being the ordinary binary DCSK.

The case (b) may have the advantage—with respect to conventional DCSK—that the signal transmitted is never repeated, thus resulting possibly in a low probability of interception (LPI).

On the other hand, Fig. 2(c,d) show the signal constellations corresponding to two versions of a four-level QCSK chaotic signaling scheme.

### 3.3. QCSK Transmitted Signal

In QCSK, similarly to DCSK, to send the symbol $s$ we transmit for half symbol period the reference signal r(t)=x(t) produced by the chaotic system and in the second half the modulated signal $m_s(t)$. The latter can be expressed as a

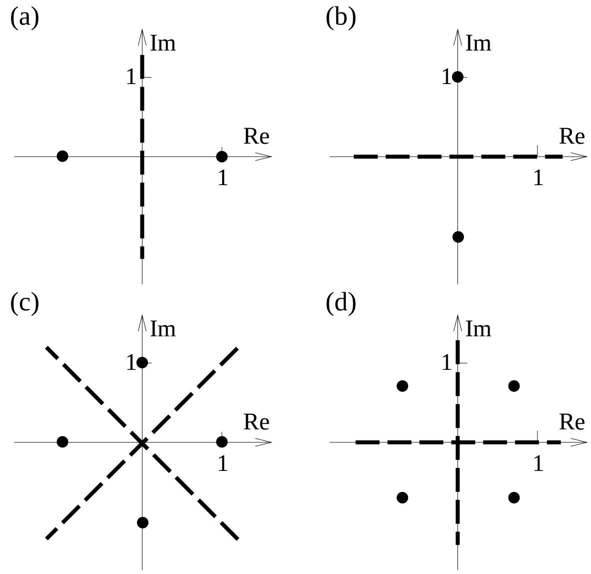


Figure 2: Chaotic constellations: (a,b) two-level signaling (DCSK), and (c,d) four-level signaling (QCSK). The dashed lines represent the decision sets.

| Chaotic signal constellations | | | |
|---|---|---|---|
| | $s$ | $c_s$ | $m_s(t)$ |
| (a) | s=1<br>s=2 | $1$<br>$-1$ | $x(t)$<br>$-x(t)$ |
| (b) | s=1<br>s=2 | $i$<br>$-i$ | $y(t)$<br>$-y(t)$ |
| (c) | s=1<br>s=2<br>s=3<br>s=4 | $1$<br>$i$<br>$-1$<br>$-i$ | $x(t)$<br>$y(t)$<br>$-x(t)$<br>$-y(t)$ |
| (d) | s=1<br>s=2<br>s=3<br>s=4 | $(+1+i)/\sqrt{2}$<br>$(-1+i)/\sqrt{2}$<br>$(-1-i)/\sqrt{2}$<br>$(+1-i)/\sqrt{2}$ | $(+x(t)+y(t))/\sqrt{2}$<br>$(-x(t)+y(t))/\sqrt{2}$<br>$(-x(t)-y(t))/\sqrt{2}$<br>$(+x(t)-y(t))/\sqrt{2}$ |

Table 1: Chaotic signal constellations and corresponding modulated signals for the cases (a), (b), (c), (d) of Fig. 2.

linear combination of the signals $x(t)$ and $y(t)$. This is illustrated in Fig. 1, where we denoted the symbol period by $T = 2\tau$.

### 3.4. QCSK Signal Detection

By correlating the modulated signal $m_s(t)$ with the reference signals $x(t)$ and $y(t)$, over $\tau = T/2$, one can retrieve the complex number $c_s = a_s + ib_s$. In fact, from (3,4) it follows that:

$$a_s = \frac{1}{P_x \tau} \int_0^\tau m_s(t)x(t)dt, \tag{6}$$

$$b_s = \frac{1}{P_x \tau} \int_0^\tau m_s(t)y(t)dt, \tag{7}$$

In practice, at the receiver we observe the noisy versions of the reference signal, $\tilde{x}(t)$, and of the modulated signal $\tilde{m}_s(t)$. By using the corrupted reference $\tilde{x}(t)$ we produce an estimate of the complementary signal $\tilde{y}(t)$ and then we correlate $\tilde{m}_s(t)$ with $\tilde{x}(t)$ and $\tilde{y}(t)$, as illustrated schematically in Fig. 1. Based on the correlation results a decision on the symbol $s$ received is taken by a decision circuit according to estimated value $\tilde{a}_s + i\tilde{b}_s$ relative to the decision sets shown in Fig. 2.

### 3.5. Extension to M-ary Constellations

In general QCSK may be extended to $M$-symbol constellations. For example, this can be obtained by considering the set of complex numbers $c_s = e^{i2s\pi/M}$, $s = 1, \ldots, M$. This choice gives a chaos-based version of $M$-ary PSK (phase shift keying).

Moreover, if the constellation signals are not restricted to lie on a circle one can design a chaotic version of QAM (quadrature amplitude modulation) [1].

## 4. NOISE PERFORMANCE

In this section we report about the performance of the proposed QCSK communication scheme in the presence of additive white gaussian noise (AWGN). The system simulated with Matlab is shown in Fig. 1, where as an example of a chaotic system we consider the 3-adic Rényi map: $f(x) = (3x+1) \mod 2 - 1$. The chaotic constellations consider are the ones shown in Fig. 2 with the corresponding decision sets. The DSP unit given $x$ computes the orthogonal signal $y$ according to the procedure described before.

Fig. 3 shows the bit error rate versus $E_b/N_0$, where $E_b$ is the energy per bit and $N_0$ is the noise power spectral density. The performance curves are plotted for the signal constellations shown in Fig. 2. One can clearly see that the BER performance is basically the same for all cases.

### 4.1. Dependence on the Correlation Time

As pointed out in [4], the performance of transmitted-reference communication schemes (such as DCSK and QCSK) depends on the averaging time $L$. This property is confirmed by Fig. 4, which shows the BER performance of the

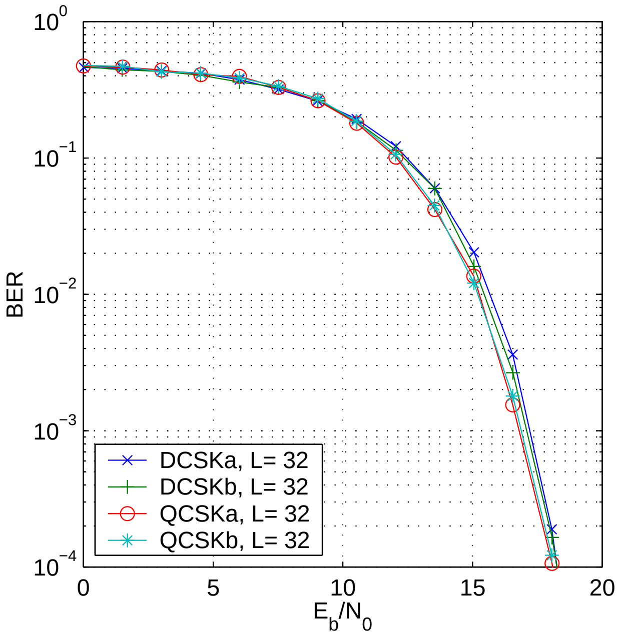


Figure 3: Comparison of BER performance: DCSK versus QCSK.

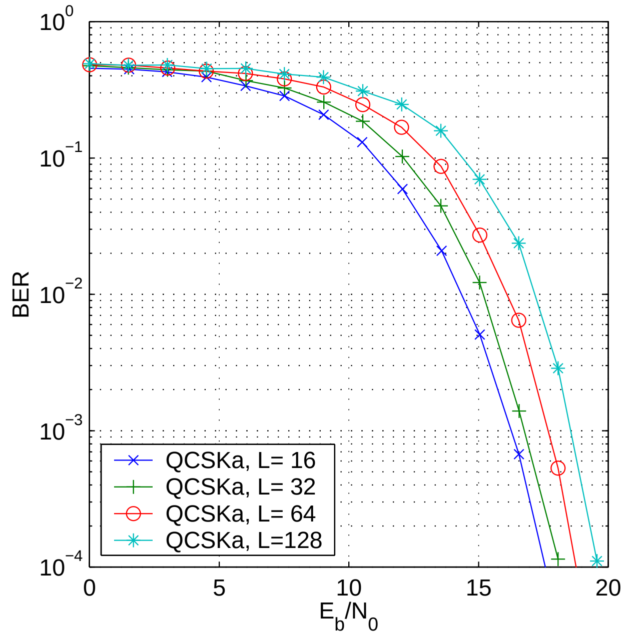


Figure 4: BER performance of QCSK for different values of the correlation time $L$.

QCSK scheme for different correlation times $L$. As visible from Fig. 4, the error probability increases as $L$ is increased.

### 4.2. QCSK versus DCSK

The QCSK may be considered equivalent to two DCSK systems: the first using the reference signal $x(t)$ and the second using the complementary signal $y(t)$ which is restored at the receiver from the reference part of the transmitted signal. The advantage of the proposed QCSK scheme is that there is no need to send the orthogonal signal over the channel. The price is the higher complexity as QCSK requires the generation of the complementary signal in both the transmitter and the receiver.

In summary, the BER performances of the DCSK and QCSK schemes are identical but QCSK has double data rate. In fact the QCSK symbol consists of two bits as opposed to one bit in DCSK. Since the two signals occupy the same bandwidth it follows that QCSK has double spectral efficiency with respect to DCSK.

## 5. CONCLUSIONS

In this paper we proposed a multilevel chaos-based modulation scheme derived from DCSK. The QCSK scheme has the advantage over DCSK of double data rate for a given bandwidth (or half bandwidth for given data rate) with the same BER performance. The drawback consists in an increased complexity of both transmitter and receiver.

## 6. REFERENCES


[1] L.W. Couch. *Digital and analog communication systems*. Prentice Hall, New Jersey, 1997.

[2] S. Hayes and C. Grebogi. Using chaos for digital communication. In T. Carroll and L. Pecora, editors, *Nonlinear Dynamics in Circuits*, pages 325–335. World Scientific, 1995.

[3] M.P. Kennedy, G. Kolumbán, G. Kis, and A.Jákó. Recent advances in communication with chaos. In *Proc. IEEE Int. Symposium on Circ. and Syst., ISCAS'98*, pages 461–464, Monterey, 1998.

[4] G. Kolumbán. Exact analytical expression for the noise performance of fm-dcsk. In *Proc. Int. Symposium on Nonlinear Theory and its Applications, NOLTA'00*, pages 735–738, Dresden, 2000.

[5] G. Kolumbán, M.P. Kennedy, and L.O. Chua. The role of synchronization in digital communication using chaos — Part II: Chaotic modulation and chaotic synchronization. *IEEE Trans. Circ. Syst.*, 45(11):1129–1140, 1998.

[6] G. Kolumbán, P. Kennedy, and G. Kis. Multilevel differential chaos shift keying. In *Proc. Int. Workshop, Nonlinear Dynamics of Electronics Systems, NDES'97*, pages 191–196, Moscow, 1997.