CLASSIFICATION OF SYNCHRONIZED STATES IN CNNS WITH HIGHER ORDER CELLS

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ABSTRACT

The aim of this paper is to describe various types of synchronization phenomena discovered in our earlier studies of dynamics of Cellular Nonlinear Networks composed of locally interconnected chaotic circuits. In our computer experiments we confirmed existence of many types of clusters of cells showing different dynamics, e.g. synchronized periodic trajectories of different types, synchronized chaotic states; different lengths of synchronized clusters. Existence of synchronization and its type depends basically on the connection strengths.

1. INTRODUCTION

Cellular Nonlinear Networks can be composed of first- or higher-order cells. They provide an universal model for a variety of phenomena observed in real physical systems. Networks of locally coupled oscillators has become an extensively studied subject in the last decade [3, 4]. Kaneko [2] introduced some basic notions to distinguish between different types of spatio-temporal behaviors. Depending on the connection type and strength of coupling a variety of interesting phenomena can be observed. This includes synchronization behavior, when all cells behave in the same manner and clustering, when some cells in the network are fully synchronized. [5, 2, 6]

Clustering is a very interesting and intriguing phenomenon. The question how and under what conditions some particular network or a part of a network of interconnected dynamical elements shows behaviors coherent in time can have very important consequences for understanding the functionality of the nervous system, ecosystems, group behavior of humans and animals or any systems composed of a large number interacting subsystems. Just considering the enormous amount of possible clustering states in a large network one can imagine its possible coding capabilities for information engineering. Clustering is also the basic phenomenon responsible for pattern formation as observed in the spatial domain in arrays of coupled systems - the network becomes ”self-organized”. Particular type of synchronization between the cells, formation of specific types of clusters is predefined by properties of the system itself.

One can influence the behavior by changing the network parameters (e.g. connection strengths) or initial conditions (which can be considered as external inputs to the individual cells).

Analytical tools are not available for cluster formation analysis we have to rely on numerical experiments. For the purpose of this study a set of software tools have been developed (see http://chopin.zet.agh.edu.pl/∼galias/nets/).

For simplicity we study the behavior of a ring of coupled chaotic oscillators. In our earlier studies we found examples of full synchronization, clustering and weak synchronization in this network. Here we discuss further details of synchronized states – synchronization of various cells is a spatial phenomenon. Looking in the time domain the signals are coherent but can represent various types of dynamics from period-1, period-2, higher-order periodic behaviors to various kinds of chaos (Roessler type 1 and type 2, double scroll etc.)

2. DYNAMICS OF THE NETWORK

Let us consider a one–dimensional CNN composed of simple third–order electronic oscillators (Chua's circuits). The circuits are coupled by means of conductances G_1. Every circuit is connected with its two nearest neighbors. The dynamics of the one–dimensional lattice composed of n circuits can be described by the following set of equations:

$$\begin{aligned} C_2\dot{x}_i &= G(z_i - x_i) - y_i + G_1(x_{i-1} - 2x_i + x_{i+1}), \\ L\dot{y}_i &= x_i, \\ C_1\dot{z}_i &= G(x_i - z_i) - f(z_i), \end{aligned} \qquad (1)$$

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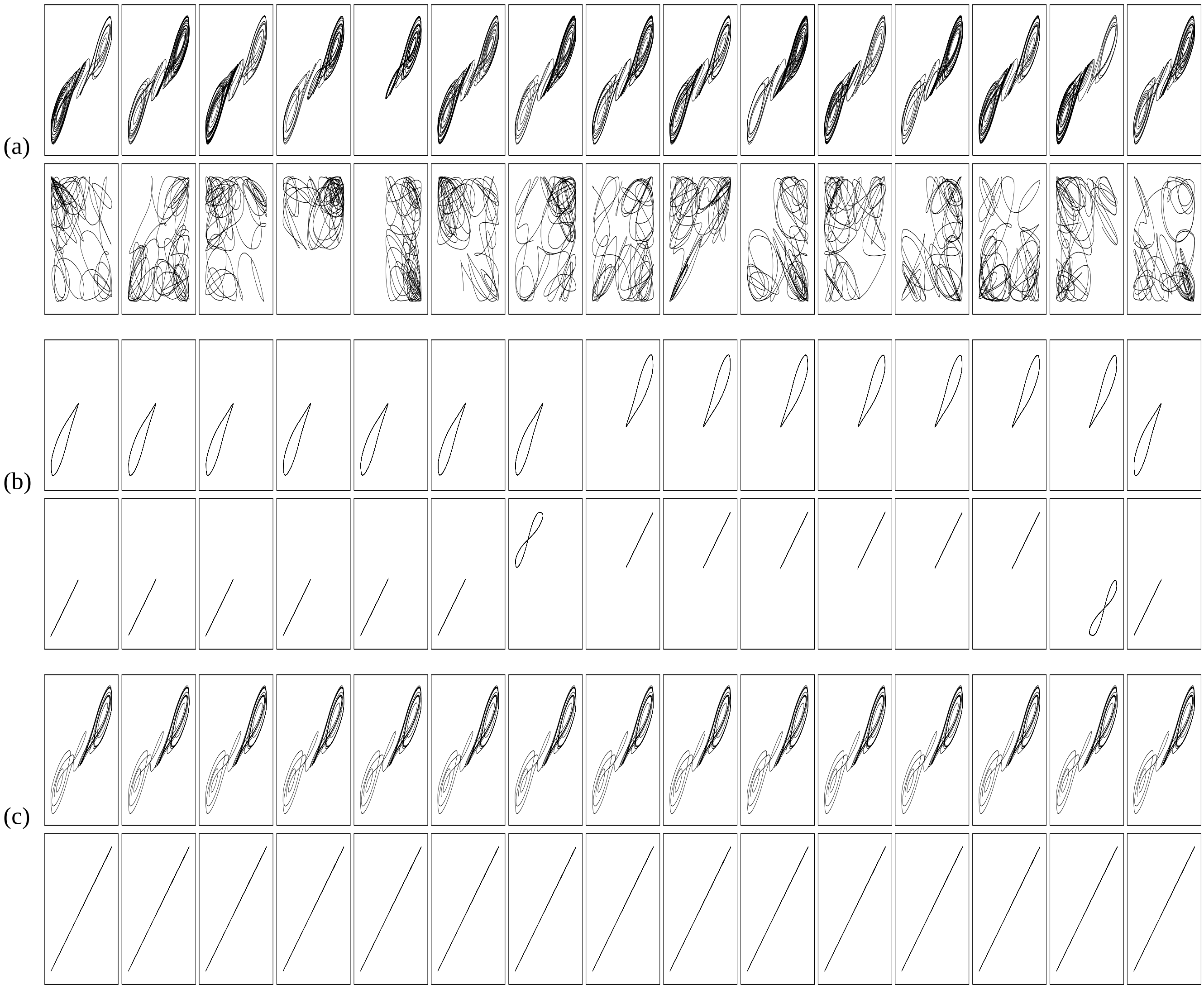

Figure 1: "Steady–state" behavior for (a) $G_1 = 0$, (b) $G_1 = 50$, (c) $G_1 = 100$

where $i = 1, 2, \ldots, n$ and where x_i and z_i denotes the voltages across the capacitances C_2 and C_1 respectively, and y_i is the current through the inductance L in the ith circuit. f is a five–segment piecewise linear function:

$$f(z) = m_2 z + \frac{m_1 - m_2}{2}(|z + B_{p_2}| - |z - B_{p_2}|) + \frac{m_0 - m_1}{2}(|z + B_{p_1}| - |z - B_{p_1}|). \quad (2)$$

The lattice forms a ring, i.e. $x_{n+1} = x_1$, $z_{n+1} = z_1$, $x_0 = x_n$, $z_0 = z_n$. In our study we use typical parameter values for which an isolated circuit generates chaotic oscillations — the "double scroll" attractor ($C_1 = 1/9$, $C_2 = 1$, $L = 1/7$, $G = 0.7$, $m_0 = -0.8$, $m_1 = -0.5$, $m_2 = 0.8$, $B_{p_1} = 1$, $B_{p_2} = 2$). In this work we consider the network composed of $n = 15$ cells.

3. SYNCHRONOUS STATES

In the first experiment we show transition from asynchronous to synchronous chaotic motion. We start the network from a point close to the synchronization space and observe its behavior for different values of coupling strength. The long term behavior (a trajectory for $t \in [1300, 1400]$) is shown in Fig. 1. For each simulation we show two lines of plots. The upper line shows individual trajectories of circuits, i.e., projection of the trajectory in a single cell onto the plane z_i, y_i. In the second line in the ith plot we show the z_i variable versus the variable z_{i+1} from the next cell. By inspecting this plot we can easily check whether neighboring cells are synchronized. In case of perfect synchronization between cells i and $i + 1$ the plot is on the diagonal line.

For $G_1 = 0$ the circuits are not coupled and hence they are oscillating independently. Each circuit forms the double–scroll attractor but their trajectories are uncorrelated.

For $G_1 = 50$ the steady state of the system becomes periodic (see Fig. 1(b)). The network consists of two clusters of cells with trajectories in the upper or lower part of the state space. In each cluster the cells are fully synchronized (diagonal lines in the second row of plots). There is a phase offset between the clusters corresponding to the "eight"-type trajectory. For $G_1 = 100$ the trajectories again are synchronized (see Fig. 1(c)) but in each cell the double–scroll attractor is formed, and all cells oscillate in a full synchrony.

We studied in more depth how the network synchronization depends on the number of coupled cells. Fig.2 gives us some insight into such a dependence. The graph shows what is the connection strength threshold value to observe full synchrony for different number of cells in the ring (5 – 15). In general the more cells are coupled in a ring the stronger the coupling must be to ensure full synchronization.

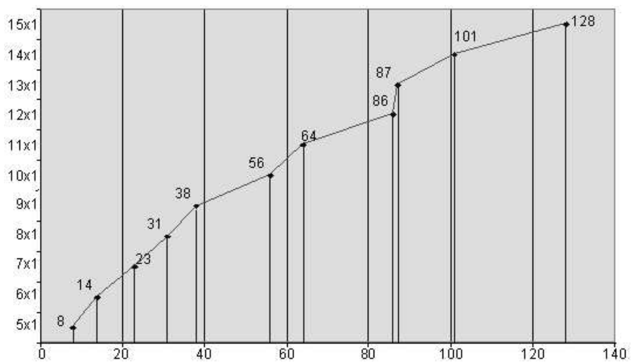

Figure 2: Dependence of the threshold value for connection strength for which full synchronization of all cells is observed on the actual length of the ring (number of cells).

4. CLUSTER TYPES

Above we have seen an example of existence of clusters in the network, where cells oscillate synchronously, although the network as a whole is not synchronized (see Fig. 1(b)). In the second part of the paper we study properties of the system in this dynamic state and investigate the process of cluster formation.

We have run a number of simulations, where the system was started from a perturbed synchronized state, i.e. the synchronous state was modified by adding a small random number to each system variable. Initially the cells were strongly connected ($G_1 = 100$). In each simulation we apply a series of changes to the coupling strength (coupling changes in time).

In the first experiment we have changed the coupling conductance to $G_1 = 50$ at $t = 30$ and back to 100 at time $t = 650$. Different unstable clusters can be observed before the steady state is observed. Initially clusters of 2 and 13 cells is formed. This structure is unstable and after a short time some cells lying on the border of the larger cluster switch the scroll and leave the cluster – one can see two clusters with sizes 5 and 10. After some more time the larger cluster decreases to have 8 cells and this state is stable in the sense that in quite a long integration time no cell changes the scroll and the cluster structure persists. The trajectory in the steady state is periodic (see Fig. 3(a)).

In Fig. 3(b) one can see the results of another simulation. This time the cluster structure is quite different. The number of clusters is much larger. Clusters have sizes 2,3 and 4 and they are separated by single cells operating in a different region of the state space. For $G_1 = 50$ in the steady state the circuits display quasi-periodic trajectory. After increasing the connection strength to $G_1 = 100$ the cluster structure is unaltered. The steady state however changes from quasiperiodic one to the period-2. The cells in the clusters become fully synchronized.

In Fig. 3(c) we show the results of simulation when we did not wait until a steady state develops for the smaller connection strength. Initially $G_1 = 100$, at $t \in [50, 70]$ connection strengths were decreased to $G_1 = 50$ and at $t = 70$ it was changed to the initial value $G_1 = 100$. In consequence for $G_1 = 50$ the steady state was not obtained and the large cluster with 12 cells survived. From the observation of the steady state for $G_1 = 100$ it follows that such a cluster is stable for this connection strength. In the steady state all cells within the cluster are fully synchronized, but contrary to the previous cases the steady state is chaotic. There is no generalized synchronization between the clusters, i.e. there is no one-to-one relation between the states (see the z_i versus z_{i+1} plot on the border of the cluster).

5. CONCLUSIONS

We have performed a series of simulations of a ring of locally connected chaotic oscillators. From these experiments one can draw several conclusions about full synchronization and cluster formation. Full synchronization is only possible for large coupling. For smaller values of coupling strength clusters are formed. Large clusters however are not stable and they loose border cells until maximum stable size is achieved. In this case cells within a cluster may be fully synchronized. Trajectories of individual cells may form periodic orbits), quasiperiodic orbits or chaotic orbits. Individual cells may oscillate periodically (period–1, see Fig. 3(a)), period–2, see Fig. 3(b)) or chaotically (see Fig. 3(c)). There exists a strong dependence between the coupling strength and the maximum size of observed stable

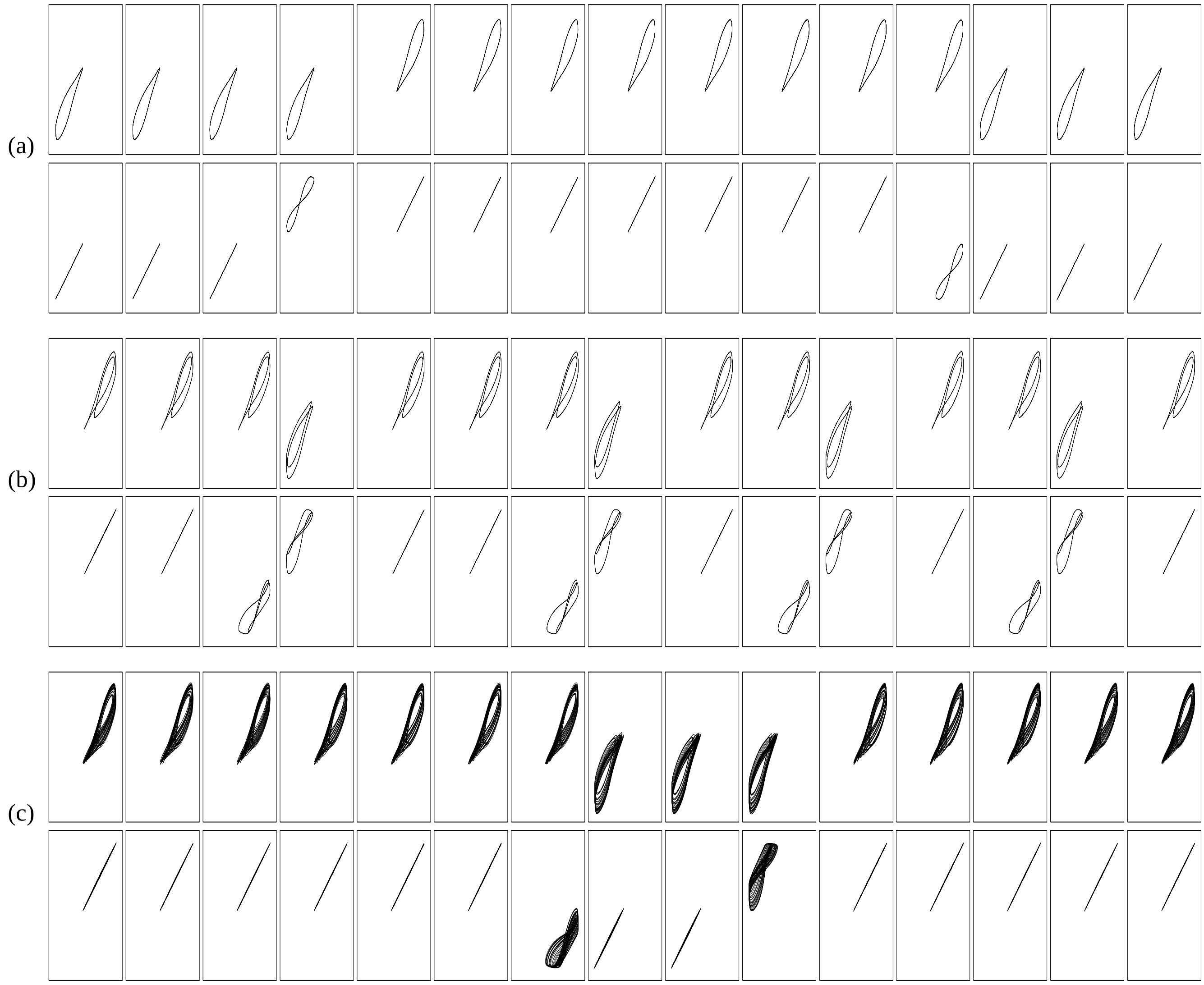

Figure 3: Different temporal behaviors within clusters. Period 1 (a), period 2 (b), Roessler-type chaos (c)

cluster. The maximum size of the stable cluster increases with the connection strength.

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