Control of a real chaotic cellular neural network

Zbigniew Galias, Josef A. Nossek

Abstract

In this paper we present the laboratory environment and experimental results of stabilization of periodic orbits in a real implementation of a chaotic three-cell cellular neural network.

1 Introduction

Several theorems on stability of cellular neural networks (CNNs) exist [1, 2], but they are rather restrictive. There are however applications, which work but for which the stable output signal for all possible input and initial patterns is not ensured. Hence one can not exclude chaotic behaviour in such networks. In this paper we study the possibilities of eliminating such behaviour by stabilizing one of existing unstable periodic orbits.

In our experiments we have used the control idea proposed by Ott *et al.* [7] called throughout the paper the OGY method. By means of this method any of the unstable periodic orbits embedded within the chaotic attractor can be stabilized and for the successful control one needs only one parameter of the system, which could be modified within some small interval around its nominal value. The idea of control is to wait until the trajectory comes close to the periodic orbit and then modify the control parameter to push the trajectory onto the stable manifold of the periodic orbit. In other words, if the actual trajectory intersects the chosen transversal plane at the point ξ_i, the approximated position of the intersection of the periodic orbit with this plane is ξ_F and the distance between ξ_i and ξ_F is smaller then some positive value d_{max}, then we alter the parameter p such that the next piercing of the plane by the trajectory ξ_{i+1} will fall onto the stable direction of the fixed point. During our experiments we have used the procedures for searching for periodic orbits, computing their Jacobians and other parameters necessary for the control described in [6].

The control procedure is applied to an autonomous three-cell CNN with the dynamics described by the following state equations:

$$\begin{aligned} \dot{x} &= -x + p_1 f(x) - s f(y) - s f(z) \\ \dot{y} &= -y - s f(x) + p_2 f(y) - r f(z) \\ \dot{z} &= -z - s f(x) + r f(y) + p_3 f(z) \end{aligned} \tag{1}$$

where $f(\cdot)$ is a saturation characteristics:

$$f(x) = 0.5(|x+1| - |x-1|) \tag{2}$$

with the following parameter set: $p_1 = 1.25$, $p_2 = 1.1$, $p_3 = 1$, $s = 3.2$, $r = 4.4$. For this set of parameters the chaotic attractor exists [3].

In section 2 the laboratory environment is presented. In section 3 the results of application of the method to controlling of a real CNN circuit are described.

2 Laboratory setup

For the experiments an electronic circuit implementing the state equations (1) has been built. We have used the circuitry proposed in [3]. The time constant of the circuit is $10ms$. We have chosen p_1 to be the control parameter. We have implemented a voltage controlled parameter by means of the MC1494 analog multiplier [4].

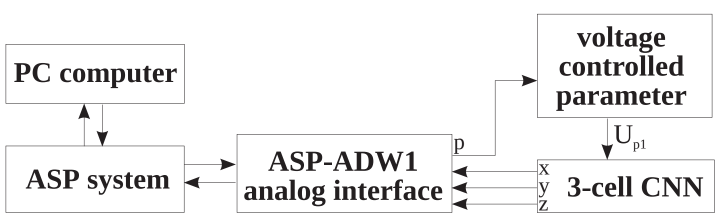

Figure 1: Hardware setup

The hardware setup is shown in Fig.1. For the acquisition of data and control we have used the ASP system. This system is equipped with 32-bit floating point NEC Advanced Signal Processor μPD77230 and works with 150 ns instruction cycle. The ASP system is connected to the analog world through the ASP-ADW1 analog interface. The analog interface supports 4 input and 1 output channels with levels $\pm$5V. The maximum sampling rate is 256kHz and the resolution of the digitized values is 12 bits.

The ASP system is also connected to a PC, which enables programming the ASP processor and interchange of data. The ASP system is used for two separate tasks, namely acquisition of data and control. For each of these tasks a separate program for the ASP system have been prepared.

The first program is devoted to the acquisition of data from the real process. The ASP processor is programmed to sample the three state variables using three input channels of the ASP-ADW1 interface with a frequency up to 240 kHz. The first state variable is used as a trigger signal. If the previous sample in the first channel is below the trigger level and the current sample is above, then samples from two other channels are written to the ASP external memory, which is visible by the host computer (PC). In this way, the PC gets a trajectory on the Poincaré plane, which is enough for computation of all control parameters. The user can choose the sampling rate and the trigger level. He can also choose the option of sending all samples, not only these lying on the Poincaré plane. A software package with graphics interface working on the PC for analyzing chaotic trajectories has been prepared. Using the procedures implemented in the software package, the parameters necessary for the control can be computed.

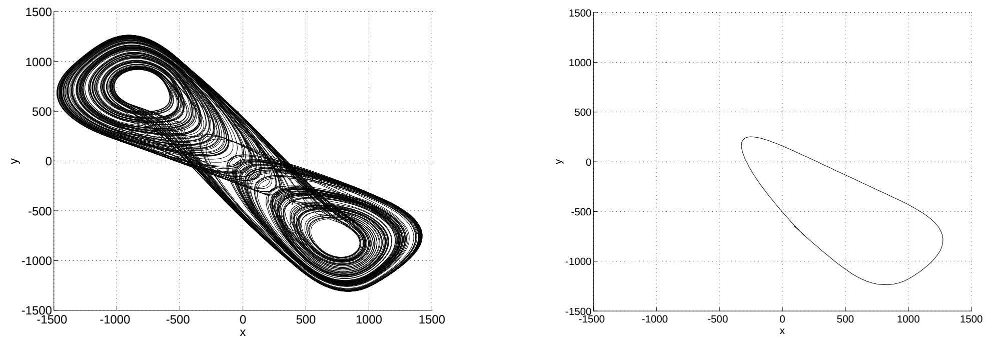

Figure 2: Chaotic trajectory and period-1 orbit recovered from experimental data

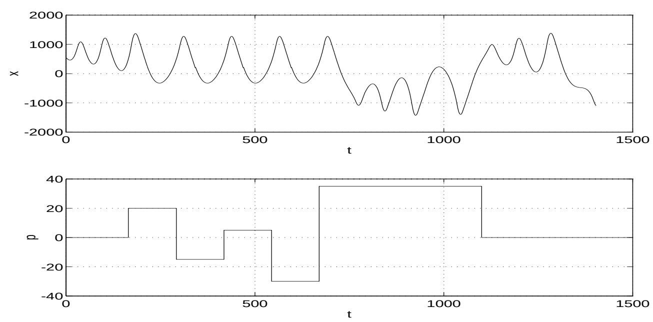

Figure 3: Control of period-1 orbit. State variable and control signal

The second ASP program is used for the control of the chaotic circuit. Three state variables are sampled. When the intersection with the transversal plane is detected, the control signal is computed and sent through the DA converter. Before sending the control signal several conditions are checked. The ASP processor checks if alteration of parameter δp is smaller in magnitude than the largest allowable parameter change δp_{max} ($|\delta p| < \delta p_{max}$) and also if the actual position on the Poincaré plane is close to the stabilized fixed point. If any of these conditions is not fullfilled, then the control value is reset ($\delta p = 0$) before sending. The host computer (PC) does not take part in the control program. The user however can modify several parameters, namely trigger level, sampling frequency, maximal allowable change of parameter and maximal distance from fixed point, for which control is still activated. After sending the control value, the coordinates on the Poincaré plane and control signal are written in the external memory so that the PC can read them and the user is able to observe the performance of the system.

We have estimated the delay between the moment, when the trajectory pierces chosen surface of section and the moment of applying the control signal to the circuit. The procedure for the detection of the intersection, computing control signal, checking all the conditions and sending the control value requires 98 processor's cycles (about $15\mu s$). Conversion time of the DAC is equal to $3\mu s$. The total delay $18\mu s$ is very small compared to the period of stabilized orbit, which is approximately $10ms$.

3 Results

An example of the chaotic trajectory obtained from the real process for a control signal $p_1 = 1.25$ ($\delta p = 0$) is shown in Fig.2. Using a specialized software package, we have found several unstable periodic orbits embedded within the attractor. The period-1 orbit, which we have tried to stabilize is shown in Fig.2. The fixed point on the Poincaré section, corresponding to this periodic orbit was found to be $\xi_F = (448.8, -957.2)^T$. In the next step, the approximation of the Jacobian of this fixed point has been computed. The eigenvalues and the corresponding eigenvectors follows: $\lambda_s = 0.335$, $\boldsymbol{e}_s = (0.863, -0.504)^T$, $\lambda_u = -3.05$, $\boldsymbol{e}_u = (0.842, 0.539)^T$. For a slightly changed control signal $\delta p = 20$ the fixed point was found to be $\xi'_F = (444.3, -959.2)^T$.

An example of the control is shown in Fig.3. One can see that for a certain period of time the trajectory remains close to the period-1 orbit, but after several periods it escapes far from it. The control signal is not reset until the next intersection with the plane $x = 500$ is detected. During computer simulations of the system (1) the stabilization of the period-1 orbit has also been very difficult. We have noticed, that trajectories starting from some points very close to the periodic orbit escape very fast into the second part of the attractor. This causes that the linearization of the behaviour of the system around periodic orbit is valid only in a very small neighbourhood of the periodic orbit. If we are not able to keep the trajectory in this small neighbourhood all the time, then control is not possible. Due to unavoidable noise existing in a real circuit, successful control is very hard to obtain. More exact results on this phenomena in an ideal 3-cell CNN system will be reported in the future.

With the used accuracy and speed of sampling we were not able to stabilize the chosen periodic orbit. There are several possible reasons for which the method does not work. One of them could be the presence of errors caused by some inaccuracies in computing the parameters of the periodic orbit, quantization and noise of circuit elements. Another reason could be strong repelling action in the unstable direction corresponding to unstable eigenvalue greater than 3 in absolute value.

In order to estimate system performance we have calculated the following expression:

$$c := \frac{1}{\#\{i : d(\xi_i, \xi_F) < d_{max}\}} \sum_{i : d(\xi_i, \xi_F) < d_{max}} \left| \frac{d(\xi_{i+1}, \xi_F)}{d(\xi_i, \xi_F)} \right| .$$

When the control program is not active, the above expression could be used as an estimation of the absolute value of the unstable eigenvalue of the periodic orbit. In this case, we have calculated this expression to be between 3 and 3.4. When the control program was active it was always smaller, namely in the interval $[2.2, 2.5]$ depending on d_{max} and the data set. Smaller value of c in the case of active control means, that repelling from the unstable periodic orbit is reduced in comparison with the uncontrolled system. The behaviour of the system is still far from the expected one (c should be close to 1 for the control to be successfull), but with the modified parameter the system trajectory stays longer in the neighbourhood of the periodic orbit, which we try to stabilize. The performance of the system is "better" when the control is active.

There are several possible modifications, which could be implemented in order to obtain a more successful control. The most important is zooming-in signals from the circuit. Although after magnification, the state variables are in the range ± 4V and use almost the whole range of AD converter we need data only from Poincaré section which exploit less than 10% of ADC range. The second problem is, that we have detected intersections of trajectories with the Poincaré plane by software. The better solution is to utilize the external clock input of the ASP-ADW1 analog interface. Intersections detected by hardware could be used as external clock to start sampling. In this way one would avoid errors caused by the finite sampling rate, which make samples on Poincaré plane noisy. Another possible modification is using different control formulas (for example modifications of OGY method proposed by Dressler and Nitsche [5]) and/or implementing multi-point control methods.

4 Conclusions

In this paper we have investigated the possibilities of supressing the chaotic behaviour in a real implementation of the three-cell CNN by stabilizing one of the unstable periodic orbits. The results obtained are promising. The data acquisition and identification part work correctly. The measured performance of the system with the active control is better (trajectory remains longer in the neighbourhood of unstable periodic orbit), than in the case of the uncontrolled circuit. From the experiments, we conclude that the OGY method is very sensitive on noise and accuracy of the computed parameters of the stabilized periodic orbit. We believe however that with some modifications described in the previous section a successful control will become possible.

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